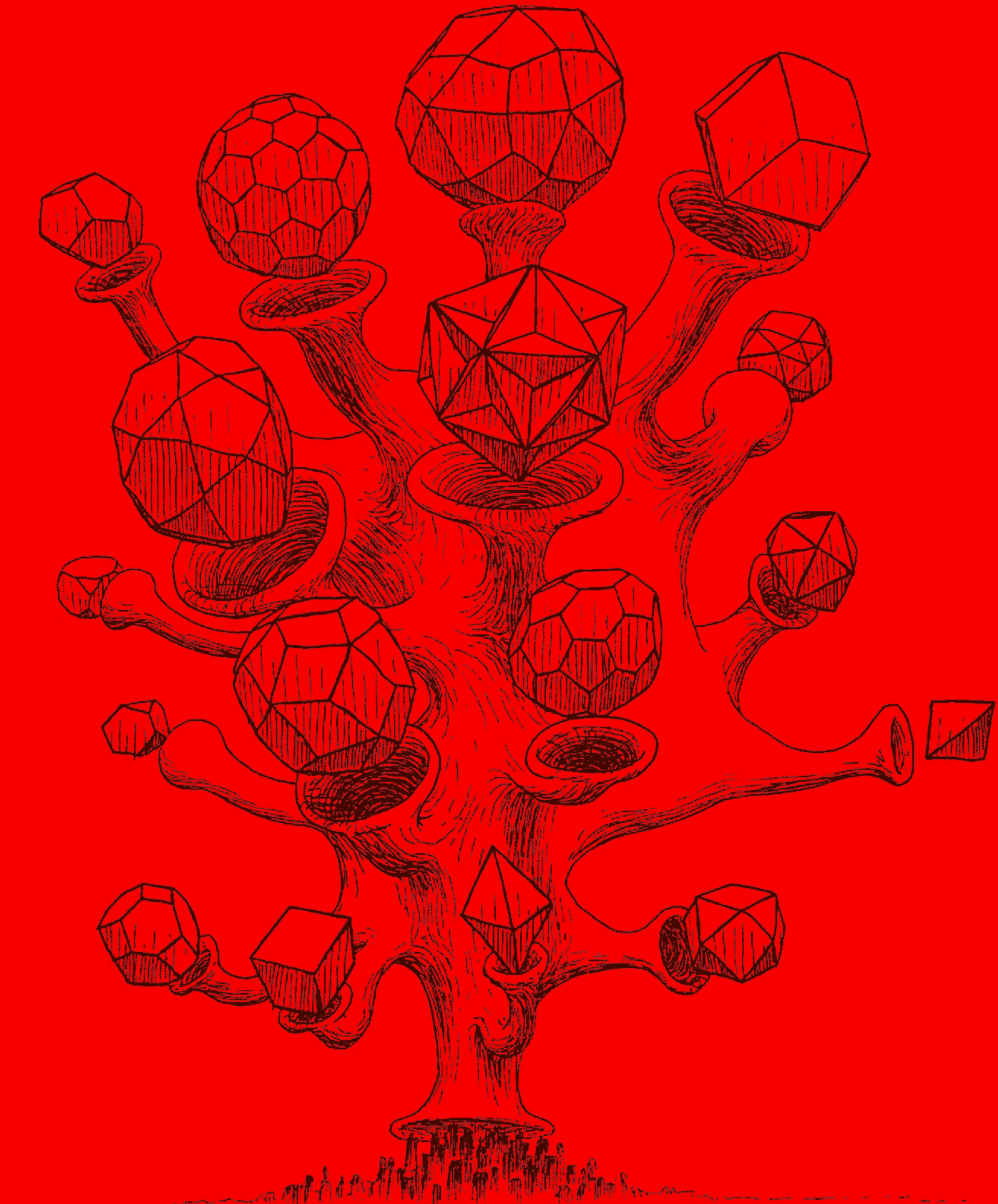
A TOM LECHNER PUBLICATION

SOLIDS



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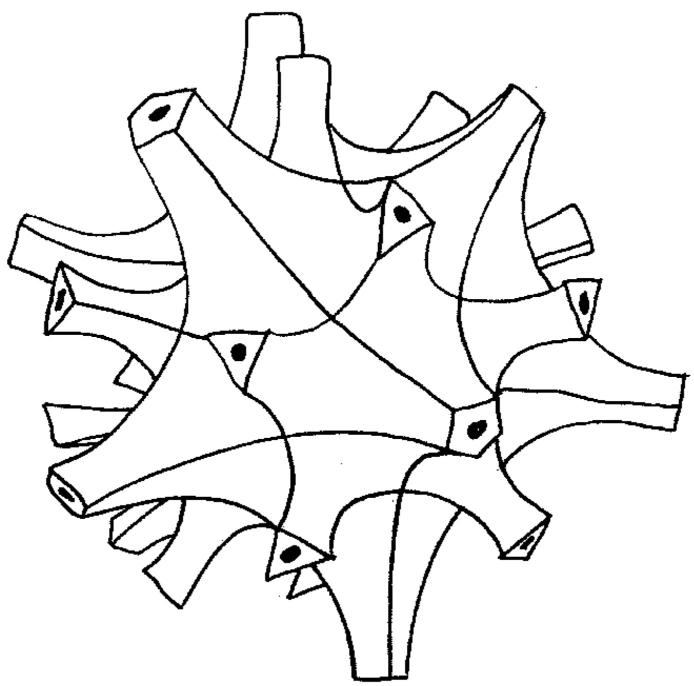

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SOLIDS

FIFTH EDITION

BY TOM LECHNER

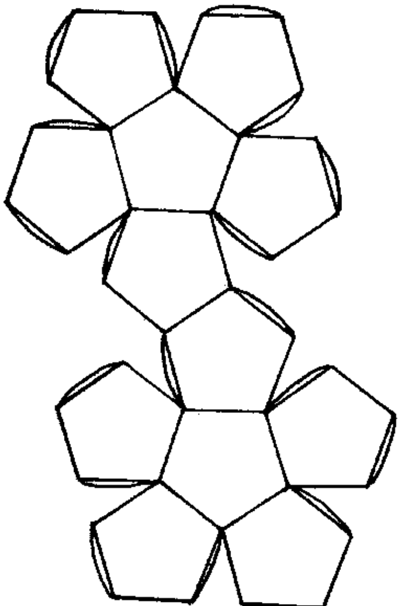
1998

FOR THOSE SHAMEFULLY KEPT FROM THE GREATER GLORY OF POLYHEDRA, AT LAST SALVATION IS AT HAND. CONTAINED HEREIN ARE ANGLES WHICH HOLD THE VERY ESSENCE OF FORM, HERETOFORE ONLY REALIZED BY MYSTICS AND MADMEN. GO FORTH, AND USE THIS KNOWLEDGE FOR GOOD.

CONTENTS

PAPER

MAKING MODELS WITH PAPER IS THE FASTEST WAY, AND PERHAPS ALSO THE MOST FLEXIBLE, TO EXPERIMENT PHYSICALLY WITH POLYHEDRA. MAYBE A CASE COULD BE MADE FOR VIRTUAL REALITY, BUT FOR OUR PURPOSES, WE MAY SAFELY DISMISS SUCH TRAPPINGS OF THE SO CALLED INFORMATION AGE. WORKING WITH PAPER NOT ONLY REFINES YOUR MANUAL DEXTERITY, BUT ALSO VIVIDLY DEMONSTRATES THE STRUCTURE OF POLYHEDRA. A LOT CAN BE LEARNED FROM TRYING TO BUILD SOMETHING.

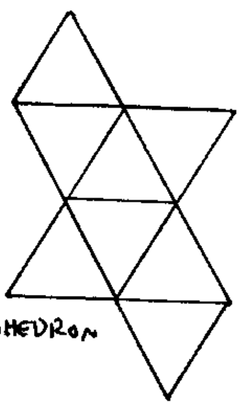


DODECAHEDRON WITH FLAPS

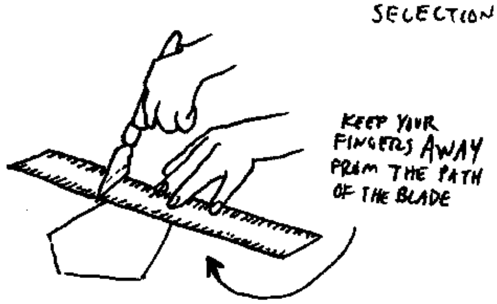
A POLYHEDRON CAN BE CONCAVE OR CONVEX. CONVEX SHAPES ARE KIND OF ROUND, AND CONCAVE SHAPES ARE MORE POINTY AND HAVE PITS, OR ONE MIGHT EVEN CALL THEM CAVES. CONVEX SHAPES, SUCH AS THE DODECAHEDRON AT RIGHT, CAN UNFOLD ONTO ONE SHEET OF PAPER WITHOUT OVERLAPPING. CONCAVE SHAPES CAN TOO, SOMETIMES, BUT IT'S TRICKIER.

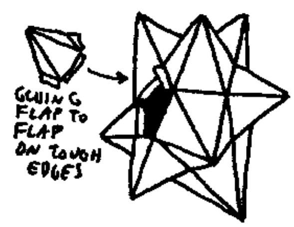
ONCE YOU HAVE A MAP FOR SOME CONVEX SHAPE, PUTTING FLAPS ON EVERY OTHER EDGE WILL ENABLE THE SHAPE TO FOLD TOGETHER WITH EACH FLAP MAGICALLY JOINING AN EMPTY EDGE. FOR CONCAVE SHAPES, GLUING FLAPS TO OTHER FLAPS CAN BE VERY HELPFUL, ESPECIALLY WHEN TRYING TO GLUE ON THE FINAL PIECE, AS IN PICTURE BELOW.

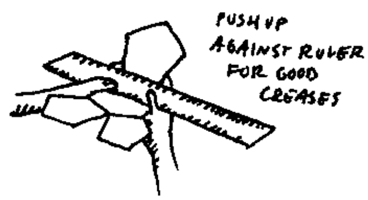
SEE THE BOOKS OF MAGNUS J. WENNINGER, ROBERT WILLIAMS, AND H. MARTIN CUNDY FOR A LARGE SELECTION OF DIAGRAMS

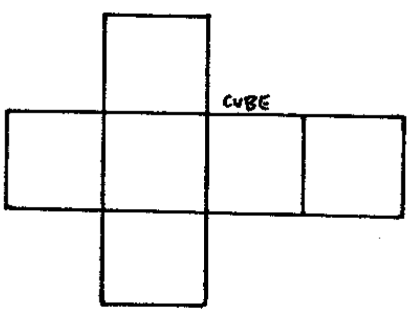


OCTAHEDRON



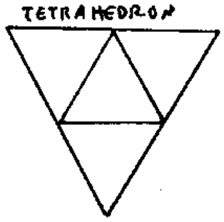




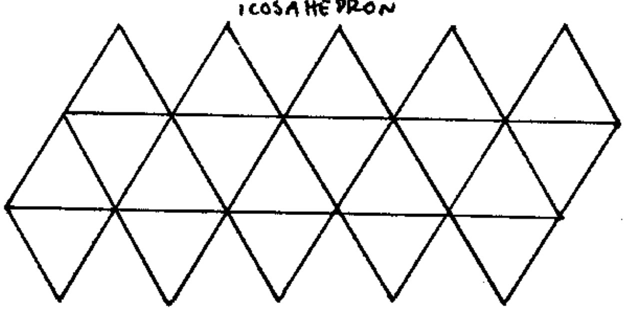


CUBE

TETRAHEDRON



ICOSAHEDRON



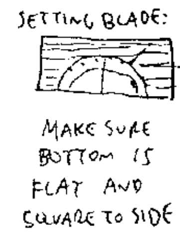
NO DOUBT YOU WILL LEARN PATIENCE AND STILLNESS, OR FORGET THE WHOLE THING ENTIRELY. IF YOU WANT THE BEAUTY OF WOOD, AND THE EASE OF PAPER, YOU COULD TRY WOOD VENEER WHICH IS SOLD IN SHEETS AT WOOD SHOPS. VENEER IS EASILY CUT AND GLUED, AND SINCE IT IS WOOD, IT CAN BE STAINED AND VARNISHED OR OTHERWISE FINISHED TO MAKE IT EXTRA PRETTY.

WOOD

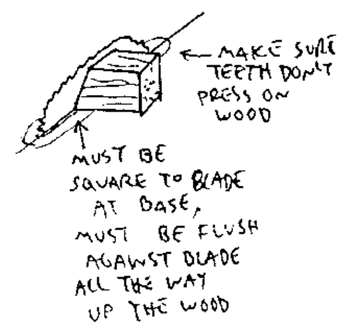
THERE HAVE BEEN WOODWORKERS SINCE TIME IMMEMORIAL. ONE OF THE MODERN RESULTS OF THIS IS THAT THERE ARE MANY BOOKS AND MAGAZINES ON THE SUBJECT, AS WELL AS MANY WOODWORKERS THEMSELVES, ALL GREAT SOURCES READY TO PROVE MY METHODS BOTH INADEQUATE AND RIDICULOUS. WHY, ONE MIGHT ASK, WOULD ANYONE WANT TO TURN THE PRECIOUS TREES INTO POLYHEDRA WHEN THEY'RE NEEDED FOR PHONE BOOKS AND APARTMENT COMPLEXES? THIS IS AN IMPORTANT QUESTION THAT YOU WILL BE ABLE TO ANSWER AFTER YOU BUY DOZENS OF COPIES OF THIS BOOK AND CONVINCE EVERYONE YOU HAVE EVER KNOWN TO DO THE SAME.

USING THE TABLE SAW

YOU WILL LEARN TO APPRECIATE ACCURACY. UNLESS YOU HAVE A FANCY-SHMANCY TABLE SAW, YOU MUST SET THE BLADE ANGLE MANUALLY. NEVER TRUST THAT STUPID PROTRACTOR ON THE SAW ITSELF. IT'S JUST THERE TO DISTRACT YOU. MEASURE WHATEVER ANGLE YOU WANT ON SOME SCRAP OF WOOD, CUT AND SAND UNTIL IT IS FLAT AND CHECK WITH PROTRACTOR UNTIL YOU ABSOLUTELY CANNOT MAKE IT BETTER.

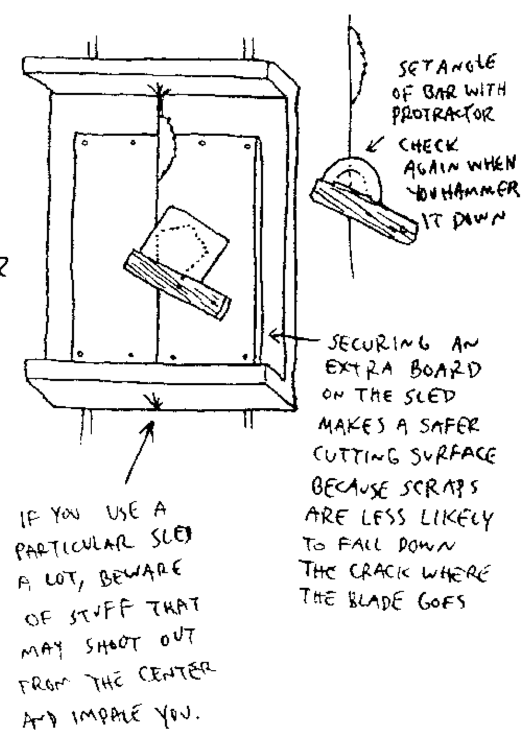


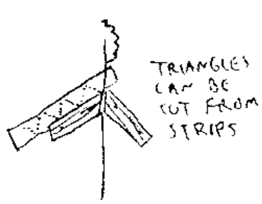
NOW YOU MUST CONSTRUCT A JIG TO CUT OUT THE SHAPES. FOR INSTANCE, FOR A PENTAGON, SET A BAR OF WOOD AT 108° TO THE LINE OF THE BLADE. AFTER ONE CUT, ROTATE THE PIECE AND LINE IT UP TO A REFERENCE MARK ON THE BAR OF WOOD. THIS MAKES THE SIDE LENGTH CONSISTENT. IF THE CUT SHAPE IS SLIGHTLY OFF AFTER THE FINAL CUT, ADJUST THE BAR ACCORDINGLY. CHECK UNTIL YOU CAN CHECK NO MORE. BETTER TO SPOT AND FIX ERRORS NOW RATHER THAN AFTER YOU CUT OUT 50 PIECES.

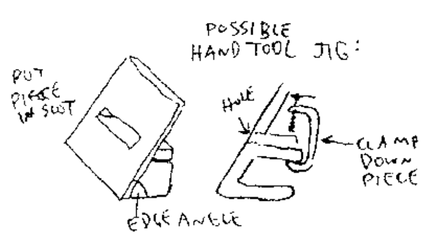


FOR SOLIDS WITH MORE THAN ONE SHAPE, BE VERY CAREFUL TO MAKE THE CORRESPONDING EDGES THE SAME. TEST AS YOU GO.

DON'T LET REPETITION DULL YOUR SENSES. WHILE YOU MAY BE ABLE TO ARRANGE YOU HANDS AT INTERESTING ANGLES AFTER YOU CUT YOUR FINGERS OFF, IT IS MORE ADVISABLE TO REST, OR GET SOMEONE ELSE TO CUT THEIR FINGERS OFF.



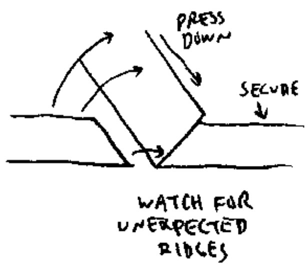




ASSEMBLING

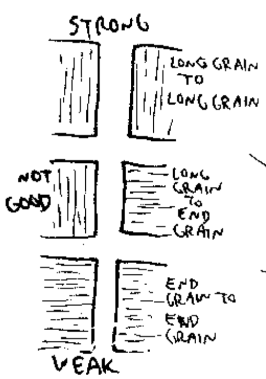
ASSUMING YOU CUT EVERYTHING ACCURATELY, THE SHAPES WILL FIT TOGETHER QUITE WELL IF YOU COULD PUT THEM ALL TOGETHER AT ONCE. UNLESS YOU ARE AN ESCAPED PRIVATE INDUSTRY BIOMECHANICAL EXPERIMENT, YOU WILL HAVE TO ASSEMBLE THE SOLID BIT BY BIT.

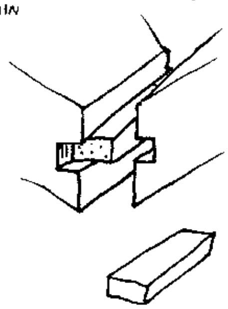
THERE ARE SURELY MANY WAYS TO ASSEMBLE THESE THINGS, AND EXPERIMENTING HELPS A LOT. I DESCRIBE HERE TWO BASIC METHODS: ONLY GLUE, GLUE AND GROOVES. BEWARE THAT JUST USING GLUE MAY ALLOW HUMIDITY OR AGE TO CONSPIRE WITH THE WOOD AND SNAP YOUR SEAMS. WITH THE GROOVY METHOD YOU CREATE A JOINT THAT HAS BOTH MECHANICAL AND CHEMICAL STRENGTH.



ONLY GLUE

IT IS OFTEN VERY DIFFICULT TO CLAMP DOWN PIECES TOGETHER TO ALLOW GLUE TO SET, SO JIGS BUILT SPECIALLY TO PROP PIECES UP MAY BE HELPFUL. FAST DRYING GLUES, SUCH AS SUPER GLUE ARE OK, IF YOU DON'T MIND TOXIC FUMES AND A WEAK JOINT. THAT TYPE OF GLUE USUALLY IS SO THIN THAT YOU CANNOT GET GOOD COVERAGE ALONG YOUR SEAM. YELLOW WOOD GLUE WORKS FAIRLY WELL. IT WILL USUALLY STICK IN ABOUT 30 SECONDS OF PRESSURE. BE GENTLE WITH THAT SEAM FOR AWHILE, THEN MOVE ON.

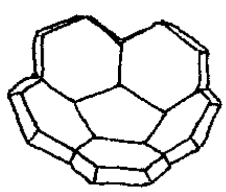




GROOVES

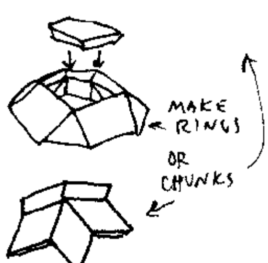
CUT GROOVES ALONG ALL THE EDGES AND THEN MAKE MANY LITTLE WOODEN BARS THAT JUST FIT IN THE GROOVES. IF ITS A TIGHT FIT, YOU MIGHT NOT EVEN NEED GLUE. WITH GLUE, REMEMBER THAT GLUE WILL GRAB THE WOOD AND MAKE IT EXPAND SLIGHTLY, SO TOO TIGHT A FIT WILL MAKE IT DIFFICULT TO PUSH YOUR PIECES TOGETHER WITHOUT BREAKING YOUR PIECES.

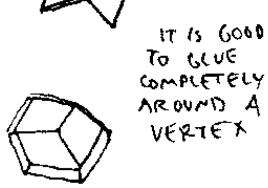
SLIDE PIECE OVER LOW BLADE. DON'T GROOVE YOUR FINGERS

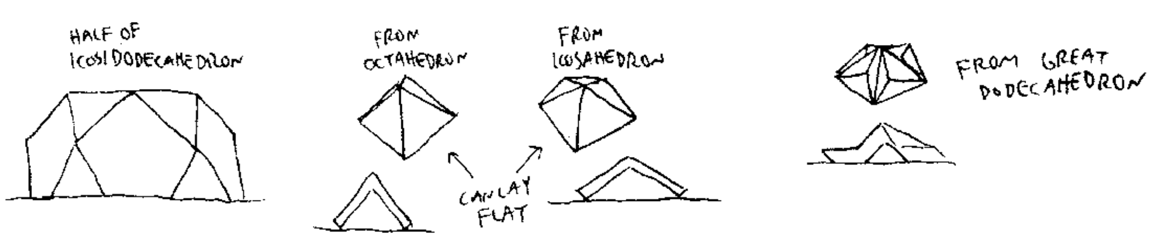


BOTH

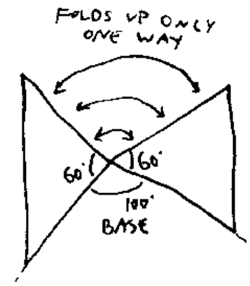
UNLESS YOU HAVE A MYSTIC CONNECTION WITH WHATEVER POLYHEDRON YOU ARE MAKING, ERRORS WILL ALWAYS CREEP IN. ONE MUST ACCEPT THIS BUT ALSO BE ON GUARD FOR IT. IF YOU BUILD UP A SHAPE ONE PIECE AT A TIME, YOU WILL BECOME FRUSTRATED TOWARD THE FINAL PIECE. IT IS BETTER TO FASTEN CHUNKS OF PIECES TOGETHER, BECAUSE THAT SPREADS THE ERRORS OVER A LARGER AREA. ONE GOOD TECHNIQUE IS TO MAKE A RING OF PIECES, WHERE POSSIBLE, THEN FASHION A PIECE TO FIT THE RESULTING HOLE. NO ONE WILL KNOW.

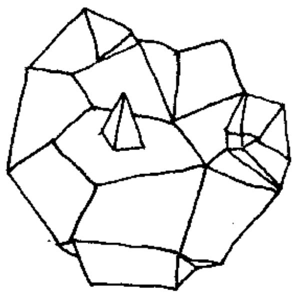






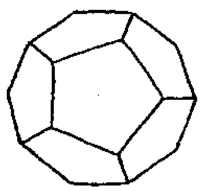
LIMITATIONS



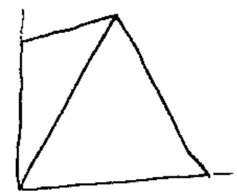


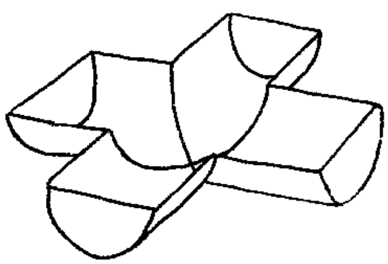
MESSY

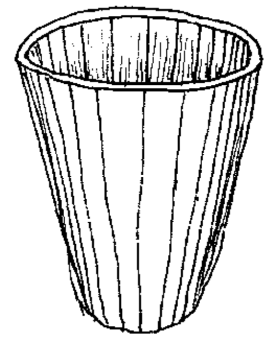
CLEAN



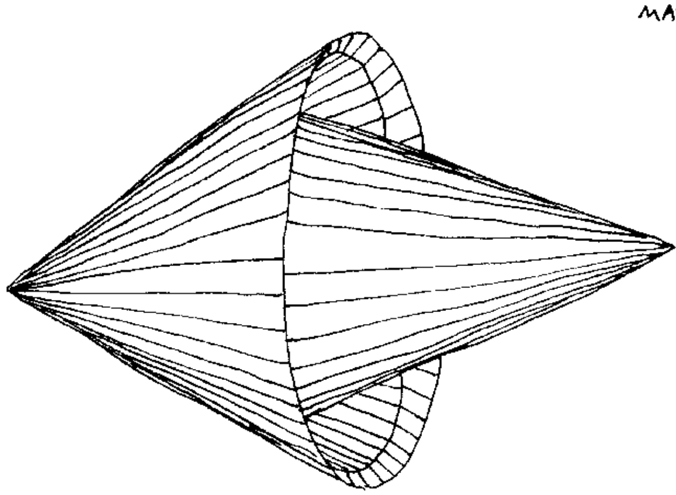
Many situations where several flat surfaces meet must obey certain relationships that are precisely described by various equations. For instance, suppose you want to build a corner that has two regular triangles (which have 60° at all corners) that sit on a base of 100°. There is only one possible orientation of these components, which means that you can compute other angles that exist in this situation, such as the angle that the triangles make with each other. Similarly, suppose you want to build a barrel of a certain height, top and bottom width, and a certain number of pieces. Those requirements demand a piece of a very particular shape, which can be readily computed.



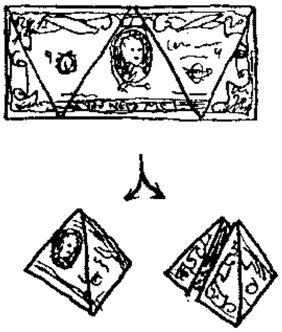




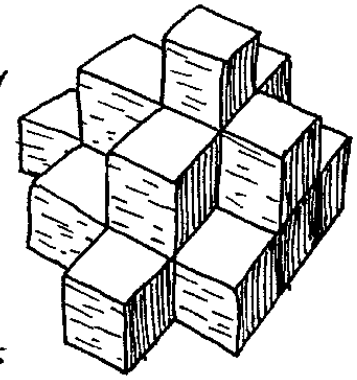
A strength of math is that if you know just a few things, often you can then figure out many other things. That could be said of many types of thought, but what's so special about math is how precise and logically consistent it all is, though it may not seem like it at first. Studying math can give you powerful methods to deal with abstract relationships. The hardest part is convincing yourself that it is in fact necessary to deal with abstract relationships. If you are convinced, perhaps you would want to try to decipher the math section so as to spot my errors for me.



FURTHER THOUGHT

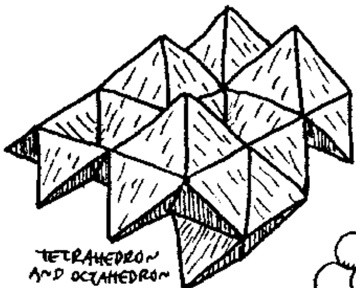


IF YOUR THOUGHTS ARE EVER TURNING TO MONEY, YOU CAN STILL LEARN A LITTLE ABOUT POLYHEDRA BY NOTICING THAT THE PROPORTIONS OF AMERICAN MONEY ALLOW IT TO BE FOLDED INTO TETRAHEDRA.

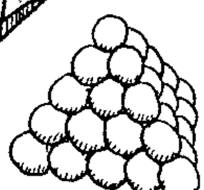


STACKING

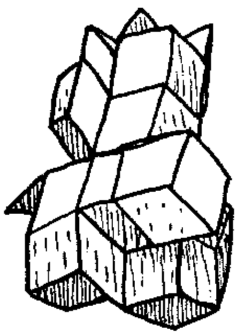
BLOCKS CAN BE STACKED ONE ON ANOTHER LEAVING NO HOLES. SHAPES BESIDES CUBES CAN DO THIS ALSO. FOR INSTANCE, THE RHOMBIC DODECAHEDRON, OR THE TETRAHEDRON TOGETHER WITH THE OCTAHEDRON. THESE ARE CLOSELY RELATED TO HOW SPHERES CAN PACK TOGETHER LIKE, FOR INSTANCE A STACK OF ORANGES. ALSO, THE GOLDEN PARALLELEPIPEDS DISCUSSED BRIEFLY IN THE MATH SECTION WILL FILL SPACE APERIODICALLY.



TETRAHEDRON AND OCTAHEDRON



ORANGES

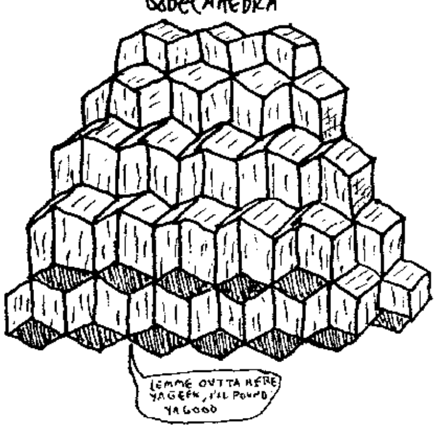


GOLDEN RHOMBOHEDRON PACKING

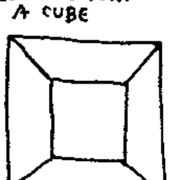
SO, IF EVER YOU WANT TO WALL SOMEONE IN SOMEWHERE, THESE SHAPES ARE THE PERFECT BRICKS, SINCE YOUR VICTIMS WILL BE SO INTRIGUED BY THE SHAPES THAT THEY WON'T REALIZE YOU'RE UP TO NO GOOD UNTIL IT'S TOO LATE.

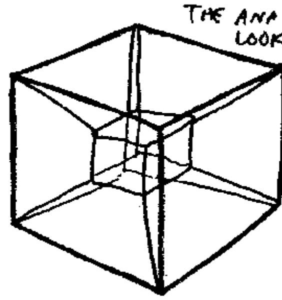
OF COURSE, YOUR POOR CAPTURED SAPS MAY ESCAPE BY STEPPING THROUGH THE FOURTH DIMENSION, OR HIGHER. WHILE THERE, THEY MAY SEE SO CALLED POLYTOPES WHICH ARE HIGHER DIMENSIONAL FORMS OF POLYHEDRA. POLYHEDRA COULD BE CALLED 3-D POLYTOPES. MOST MATHEMATICIANS WILL TELL YOU THERE'S NOTHING SPECIAL ABOUT HIGHER DIMENSIONS, BUT DON'T YOU BELIEVE THEM. THEY ARE SILENT BECAUSE THEY ARE SWORN TO SECRECY. LUCKY FOR THE REST OF US THERE ARE THOSE WHO AREN'T AFRAID TO SPEAK THE TRUTH.

RHOMBIC DODECAHEDRA



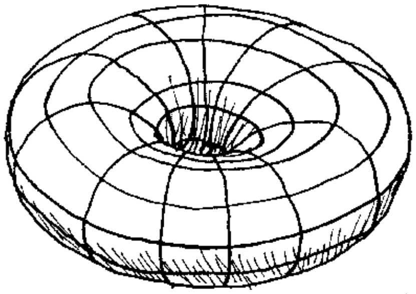
LOOKING DOWN A CUBE



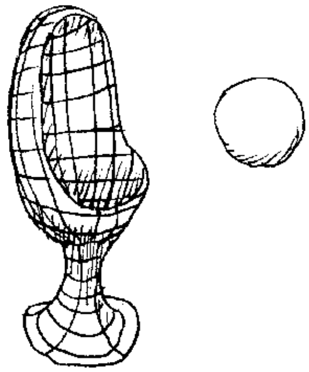


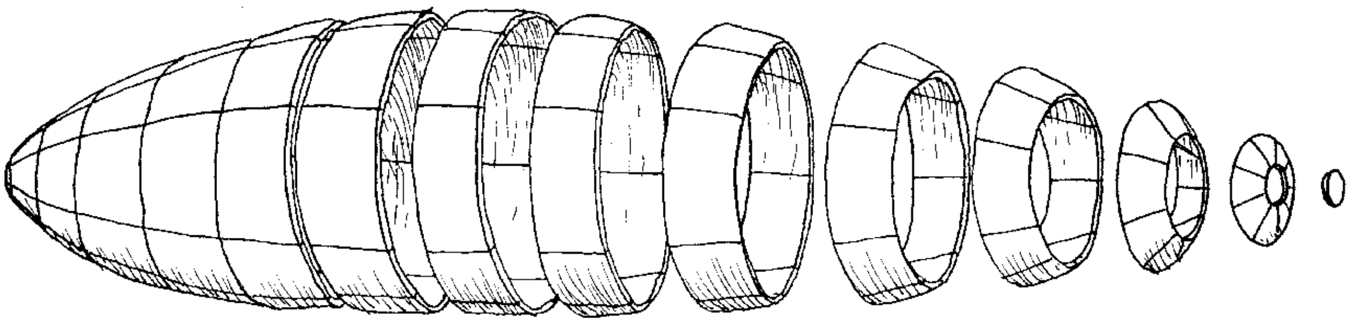
THE ANALOGOUS LOOKING DOWN A HYPERCUBE (A 4-DIMENSIONAL CUBE)

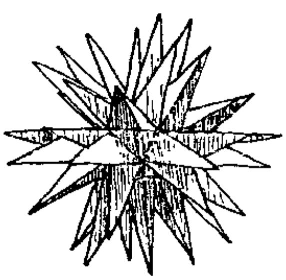
SOLIDS OF ROTATION



WHAT PRIMATIVE 3-D MODELLING PROGRAMS DO BADLY, YOU CAN DO WITH PANACHE IN THE REAL WORLD, WITH FULLY DIMENSIONAL PRODUCTS OF NATURE, THAT IS TO SAY, DEAD TREES. THEIR DEATHS WOULD NOT BE IN VAIN! WORK WITH NATURE, OR BE A SLAVE TO HIGH TECHNOLOGY. COME ON, WHAT'S IT GOING TO BE?



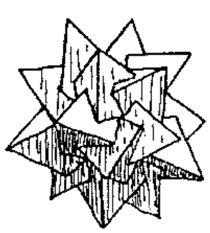


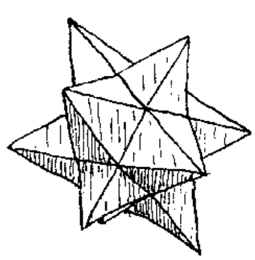


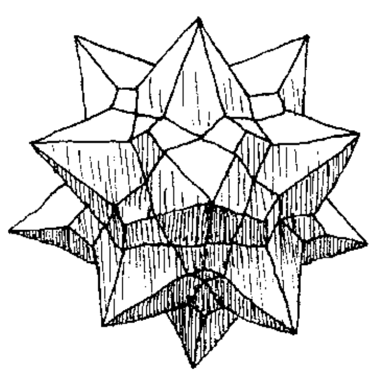
STELLATIONS AND COMPOUNDS

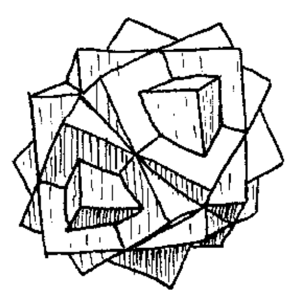
A FRUITFUL WAY TO GENERATE NEW FORMS IS BY PUTTING TOGETHER OLD ONES, THEREBY GENERATING A COMPOUND. ANOTHER WAY IS TO STELLATE. STELLATING MEANS EXTENDING THE FACES OF A SOLID SO AS TO ENCLOSE MORE SPACE. THE MORE SIDES A SOLID HAS, THE MORE STELLATIONS YOU CAN PLUNDER. I HEAR TALE THE ICOSAHEDRON ALONE HAS 59 STELLATIONS.

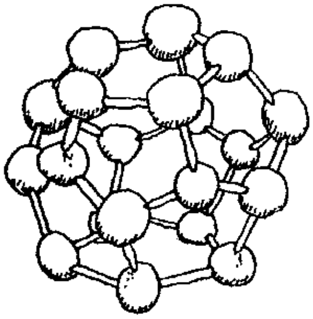
MANY STELLATIONS TURN OUT TO BE COMPOUNDS OF SIMPLER SHAPES. THE ONLY PROBLEM WITH STELLATIONS IS THAT YOU COULD RAISE SEVERAL CHILDREN IN THE TIME IT TAKES TO FIGURE THEM OUT. THE BEST STRATEGY IS TO BEGET A GENIUS, THEREBY CELEBRATING THE JOYS OF LIFE **AND** HAVING SOMEONE ELSE DO YOUR WORK FOR YOU





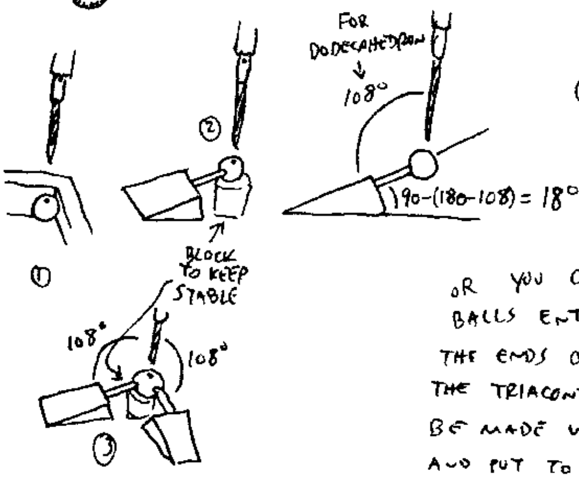


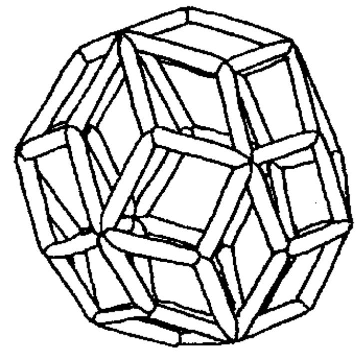




IF YOU DON'T WANT TO HAVE FLAT SIDED THINGS CLUTTERING UP YOUR ENVIRONMENT, YOU COULD MAKE BALL AND STICK MODELS OR DOWEL FRAMES THAT CLUTTER UP YOUR ENVIRONMENT.

FOR EXAMPLE, BALLS FOR A DODECAHEDRON CAN BE MADE AS IN THE DIAGRAM. ① DRILL ONE HOLE
② STICK ON PEG ELEVATED (FACE ANGLE) − 90 DEGREES, DRILL SECOND HOLE
③ STICK ANOTHER SORT OF WEDGE, WOBBLE UNTIL FLAT, DRILL THIRD HOLE



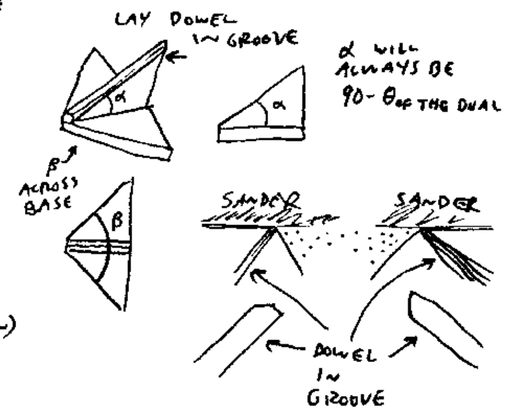


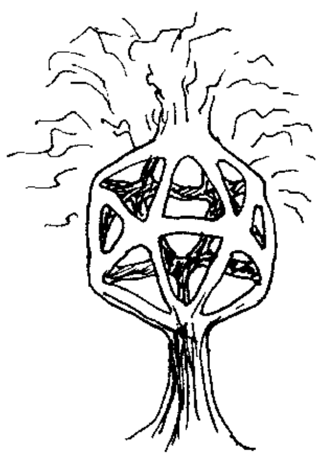
OR YOU COULD DO AWAY WITH BALLS ENTIRELY AND SHAVE OFF THE ENDS OF DOWELS. FOR INSTANCE THE TRIACONTAHEDRON DOWELS CAN BE MADE WITH THE JIG AT RIGHT, AND PUT TO THE SANDER. FOR ONE END OF THE RODS, WHERE 5 COME TOGETHER,

$\alpha = 90 - (\theta_P$ OF ICOSIDODECAHEDRON$)$
$\beta = 72$

AND WHERE 3 COME TOGETHER

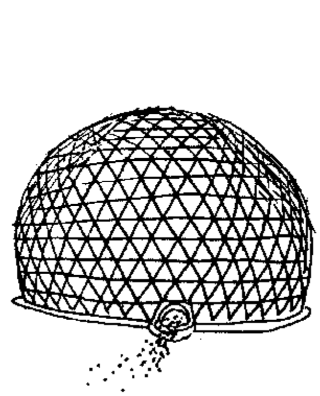
$\alpha = 90 - (\theta_T$ OF ICOSIDODECAHEDRON$)$
$\beta = 120$



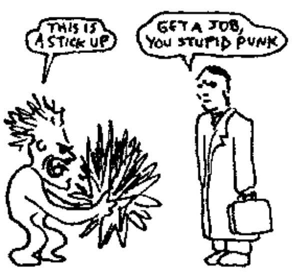


PLANT TREES AROUND GUIDES

NO NEED TO STOP WITH WOOD! THERE'S ALWAYS STAINED GLASS, STAINLESS STEEL, THE LATEST CARBON COMPOSITES, PLASTER, MOLECULAR ENGINEERING, MICROPROCESSOR DESIGN, CONCRETE, ANT FARMS, PORCELAIN, STONE AND FABRIC. THE POSSIBILITIES ARE ENDLESS, LIMITED ONLY BY COST, AVAILABILITY OF MATERIALS, OBEDIENT LABOR, ENVIRONMENTAL IMPACT, GENETIC PREDISPOSITION, TIME, WEATHER, TOOLS, HEALTH, AND SAFETY!



CONCERT HALLS



WEAPONS

CONVERSATION PIECE

FURTHER READING

IF YOU ARE INSULTED BY MY TAWDRY AMATEUR THEATRICS, HERE ARE SOME SOURCES THAT MAY LEAD YOU ON BETTER.

W.W. ROUSE BALL, H.S.M. COXETER, MATHEMATICAL RECREATIONS AND ESSAYS, DOVER 1987

H.S.M. COXETER, INTRODUCTION TO GEOMETRY, JOHN WILEY & SONS INC. 1961, 1969
REGULAR POLYTOPES, DOVER 1973
REGULAR COMPLEX POLYTOPES, CAMBRIDGE UNIVERSITY PRESS, 1974, 1991
- IF YOU WANT THE ROBUST MATHEMATICAL THEORY IN NOT JUST 3 DIMENSIONS, THESE ARE A VERY GOOD START.

H.S.M. COXETER, H.T. FLATHER, J.F. PETRIE, P. DUVAL, THE 59 ICOSAHEDRA, SPRINGER-VERLAG 1938

H.S.M. COXETER, M.S. LONGUET-HIGGINS, J.C.P. MILLER, "UNIFORM POLYHEDRA," PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY, SERIES A, VOL 246 (1954) P401-449
- THESE LAST TWO ARE A WEALTH OF DIAGRAMS AND DESCRIPTIONS. "UNIFORM POLYHEDRA" IS THE FIRST ANNOUNCEMENT TO THE WORLD OF THE WHOLE LIST OF UNIFORM POLYHEDRA PROVEN COMPLETE IN 1975 BY J. SKILLING (SEE REFERENCE BELOW).

PETER CROMWELL, POLYHEDRA, CAMBRIDGE UNIVERSITY PRESS, 1997
- INFORMATIVE AND EVEN USEFUL OVERVIEW OF THE MULTIFACETED HISTORY AND NATURE OF POLYHEDRA. LOTS OF DIAGRAMS AND REFERENCES. ADMIRABLE AT LEAST FOR ITS WEIGHT.

H. MARTIN CUNDY, A.P. ROLLET, MATHEMATICAL MODELS, OXFORD UNIVERSITY PRESS, 1961
- THIS BOOK CONTAINS A GOOD TABLE OF SOLIDS WITH LOTS OF NUMERIC INFORMATION, PLUS A LOT OF OTHER MATERIAL ABOUT HOW TO DEMONSTRATE VARIOUS MATH PRINCIPLES. HIGHLY RECOMMENDED FOR THE MATH MODEL MAKER.

ANIELA EHRENFUCHT, THE CUBE MADE INTERESTING, PERGAMON PRESS 1964
- AND WHAT BETTER WAY THEN RED AND BLUE 3-D PICTURES? (COMES WITH GLASSES)

ISTVÁN AND MAGDOLNA HARGITTAI, SYMMETRY THROUGH THE EYES OF A CHEMIST, 2nd EDITION, PLENUM PRESS 1995
SYMMETRY, SHELTER PUBLICATIONS INC. 1994
- THESE ARE FILLED WITH PICTURES OF SYMMETRY FROM AROUND THE WORLD.

ALAN HOLDEN, SHAPES, SPACE, AND SYMMETRY, COLUMBIA UNIVERSITY PRESS 1971, DOVER 1991
ORDERLY TANGLES COLUMBIA UNIV. PRESS 1983
- GOOD USE OF ABUNDANT PHOTOS. TANGLES HAS MANY INTERESTING INTERTWINING DOWEL CONSTRUCTIONS

FELIX KLEIN, LECTURES ON THE ICOSAHEDRON AND THE SOLUTION OF EQUATIONS OF THE FIFTH DEGREE, DOVER 1956
- HE ISN'T KIDDING.

L. LINES, SOLID GEOMETRY, DOVER 1965
- CONTAINS MANY USEFUL THEOREMS, PLUS A BIG SECTION ON POLYHEDRA.

J. SKILLING. "THE COMPLETE SET OF UNIFORM POLYHEDRA," PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY, SERIES A, VOL 278 (1975) P 111-135

MAGNUS J. WENNINGER, POLYHEDRON MODELS, CAMBRIDGE UNIVERSITY PRESS, 1973
DUAL MODELS, CAMBRIDGE 1983
SPHERICAL MODELS, CAMBRIDGE 1979
- IF NOTHING ELSE YOU WILL LEARN COMPLICATION FROM THESE. THEY CONTAIN A LARGE ARRAY OF QUITE COMPLICATED POLYHEDRA, AND CONSTRUCTION INSTRUCTIONS. HIGHLY RECOMMENDED.

ROBERT WILLIAMS, THE GEOMETRICAL FOUNDATION OF NATURAL STRUCTURE, DOVER 1979
- MANY SCIENCE REFERENCES. HAS A TABLE OF SOLIDS (WITH NETS) AND A TABLE OF SOLID PACKINGS. MUCH ABOUT AGGREGATIONS AND TIPS FOR DEVELOPING NEW FORMS.

VIKTOR A. ZALGALLAR, CONVEX POLYHEDRA WITH REGULAR FACES, CONSULTANTS BUREAU, NY 1969
- A PROOF THAT ALL CONVEX REGULAR FACED POLYHEDRA CAN BE BUILT FROM 28 BASIC SHAPES PLUS THE PRISMS AND ANTIPRISMS

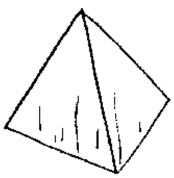
TABLE OF SOLIDS

IF YOU REALLY NEED MORE PRECISION, CONSULT THE MATH SECTION.

θ IS THE ANGLE ALONG AN EDGE BETWEEN THE PLANE OF THE FACE AND THE PLANE CONNECTING THE EDGE TO THE SOLID'S CENTER, MEASURED INSIDE. $\bar{\theta}$ IS MEASURED OUTSIDE.

δ IS A DIHEDRAL ANGLE, SO FOR INSTANCE δ_{HT} IS THE ANGLE BETWEEN A HEXAGON AND A TRIANGLE, δ_2 IS THE ANGLE ALONG AN EDGE S_2. $\bar{\delta}$ IS MEASURED OUTSIDE

$\frac{I}{S}, \frac{M}{S}, \frac{O}{S}$ ARE RATIOS THAT TELL HOW BIG A SOLID IS COMPARED TO AN EDGE. THUS, SAY YOU WANT A TRUNCATED OCTAHEDRON ABOUT 50cm WIDE. THE RATIO $\frac{O}{S} = 3.162$ SO YOU NEED PIECES THAT HAVE EDGE LENGTH $S = \frac{50\text{cm}}{3.162} = 15.81$ cm



TETRAHEDRON

4 TRIANGLES
4 VERTICES
6 EDGES

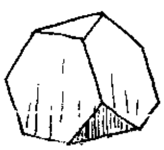
$\boxed{\theta_T = 35.26438968}$

$\delta_{TT} = 70.52877937$

$$\frac{\text{(INNER SPHERE DIAMETER)}}{\text{(SIDE LENGTH)}} = .4082482905$$

$$\frac{\text{(MID-EDGE SPHERE DIAMETER)}}{\text{(SIDE LENGTH)}} = .7071067812$$

$$\frac{\text{(OUTER SPHERE DIAMETER)}}{\text{(SIDE LENGTH)}} = 1.224744871$$



TRUNCATED TETRAHEDRON

4 HEXAGONS
4 TRIANGLES
12 VERTICES
18 EDGES

$\boxed{\theta_H = 35.26438968}$

$\boxed{\theta_T = 74.20683095}$

$\delta_{HH} = 70.52877937$

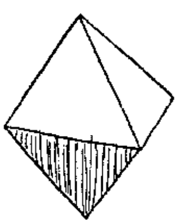
$\delta_{HT} = 109.4712206$

$\frac{I_H}{s} = 1.224744871$

$\frac{I_T}{s} = 2.041241452$

$\frac{M}{s} = 2.121320344$

$\frac{O}{s} = 2.345207880$



OCTAHEDRON

8 TRIANGLES
6 VERTICES
12 EDGES

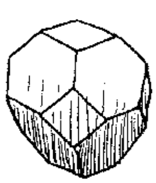
$\boxed{\theta_T = 54.73561032}$

$\delta_{TT} = 109.4712206$

$\frac{I}{s} = .8164965809$

$\frac{M}{s} = 1$

$\frac{O}{s} = 1.414213562$



TRUNCATED OCTAHEDRON

8 HEXAGONS
6 SQUARES
24 VERTICES
36 EDGES

$\boxed{\theta_H = 54.73561032}$

$\boxed{\theta_S = 70.52877937}$

$\delta_{HH} = 109.4712206$

$\delta_{HS} = 125.2643897$

$\frac{I_H}{s} = 2.449489743$

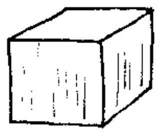
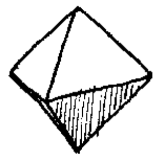
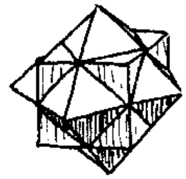
$\frac{I_S}{s} = 2.828427125$

$\frac{M}{s} = 3$

$\frac{O}{s} = 3.16227766$

FOR THIS AND THE NEXT 3 PAIRS OF PAGES, EACH SOLID IS WHAT'S CALLED THE DUAL OF THE SOLID ACROSS ON THE OPPOSITE PAGE. DUALS HAVE THE SAME NUMBER OF EDGES, BUT THE NUMBER OF CORNERS AND FACES IS INTERCHANGED.

A SOLID AND ITS DUAL MAY BE COMPOUNDED SO THAT THEIR EDGES TOUCH BY REQUIRING THEY HAVE THE SAME MID-EDGE SPHERE DIAMETER. THIS SPHERE, CALLED THE INTERSPHERE IS TANGENT TO ALL THE EDGES OF BOTH SOLIDS.

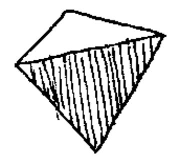

AND

MAKE


FOR EXAMPLE,

$$M = \left(\frac{M}{S}\right)_{OCT} S_{OCT} = \left(\frac{M}{S}\right)_{CUBE} S_{CUBE}$$

SO $$\frac{S_{OCTAHEDRON}}{S_{CUBE}} = \frac{(M/S)_{OCT}}{(M/S)_{CUBE}} = \sqrt{2}$$

TETRAHEDRON IS DUAL TO ITSELF



$\frac{I}{S_1} = .639602149$

$\frac{M}{S_1} = .707106781$

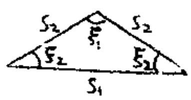
$\frac{O_3}{S_1} = .734846923$

$\frac{O_6}{S_1} = 1.224744871$

$\frac{S_1}{S_2} = 1\frac{2}{3}$

$\boxed{\theta = 64.76059818}$

$\delta = 129.5211964$



$\xi_1 = 112.8853805$

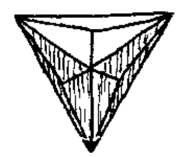
$\xi_2 = 33.55730976$

TRIAKIS TETRAHEDRON

12 ISOSCELES TRIANGLES

8 VERTICES

18 EDGES



$\frac{I}{S} = 1$

$\frac{M}{S} = 1.414213562$

$\frac{O}{S} = 1.732050808$

$\boxed{\theta = 45}$

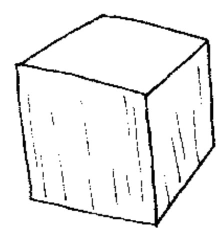
$\delta = 90$

CUBE

6 SQUARES

8 VERTICES

12 EDGES



$\frac{I}{S_1} = 1.341640787$

$\frac{M}{S_1} = 1.414213562$

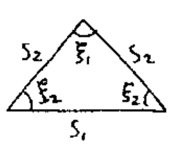
$\frac{O_4}{S_1} = 1\frac{1}{2}$

$\frac{O_6}{S_1} = 1.73205081$

$\frac{S_1}{S_2} = 1\frac{1}{3}$

$\boxed{\theta = 71.56505118}$

$\delta = 143.1301024$



$\xi_1 = 83.62062979$

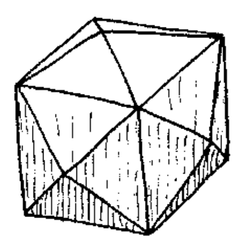
$\xi_2 = 48.18968510$

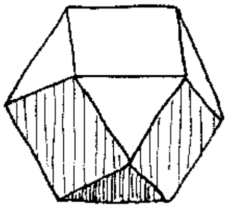
TETRAKIS HEXAHEDRON

24 ISOSCELES TRIANGLES

14 VERTICES

36 EDGES





CUBOCTAHEDRON

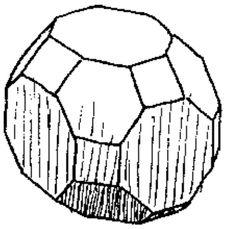
6 SQUARES
8 TRIANGLES

12 VERTICES
24 EDGES

$\boxed{\theta_S = 54.73561032}$
$\boxed{\theta_T = 70.52877937}$

$\delta_{ST} = 125.2643897$

$\frac{I_S}{s} = 1.414213562$
$\frac{I_T}{s} = 1.632993162$
$\frac{M}{s} = 1.732050808$
$\frac{O}{s} = 2$



TRUNCATED CUBOCTAHEDRON

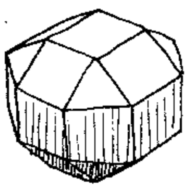
6 OCTAGONS
8 HEXAGONS
12 SQUARES

48 VERTICES
72 EDGES

$\boxed{\theta_O = 57.76438968}$
$\boxed{\theta_H = 67.5}$
$\boxed{\theta_S = 77.23561032}$

$\delta_{OH} = 125.26438968$
$\delta_{OS} = 135$
$\delta_{HS} = 144.73561032$

$\frac{I_O}{s} = 3.828427125$
$\frac{I_H}{s} = 4.18154055$
$\frac{I_S}{s} = 4.414213562$
$\frac{M}{s} = 4.526066877$
$\frac{O}{s} = 4.635221826$



RHOMBICUBOCTAHEDRON

18 SQUARES
8 TRIANGLES

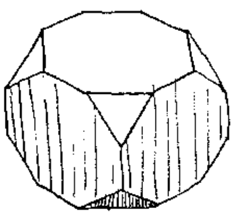
24 VERTICES
48 EDGES

$\boxed{\theta_S = 67.5}$
$\boxed{\theta_T = 77.23561032}$

$\delta_{SS} = 135$

$\delta_{ST} = 144.73561032$

$\frac{I_S}{s} = 2.414213562$
$\frac{I_T}{s} = 2.548547388$
$\frac{M}{s} = 2.61312593$
$\frac{O}{s} = 2.797932652$



TRUNCATED CUBE

6 OCTAGONS
8 TRIANGLES

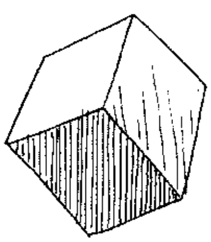
24 VERTICES
36 EDGES

$\boxed{\theta_O = 45}$
$\boxed{\theta_T = 80.26438968}$

$\delta_{OO} = 90$

$\delta_{OT} = 125.26438968$

$\frac{I_O}{s} = 2.414213562$
$\frac{I_T}{s} = 2.956795679$
$\frac{M}{s} = 3.414213562$
$\frac{O}{s} = 3.557647291$



CUBE

6 SQUARES

8 VERTICES
12 EDGES

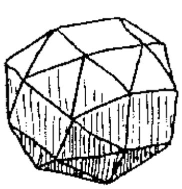
$\boxed{\theta_S = 45}$

$\delta_{SS} = 90$

$\frac{I}{s} = 1$

$\frac{M}{s} = 1.414213562$

$\frac{O}{s} = 1.732050808$



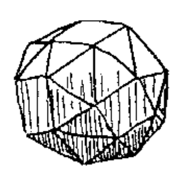
SNUB CUBE

6 SQUARES
32 TRIANGLES

24 VERTICES
60 EDGES

$\boxed{\theta_S = 66.36613621}$
$\boxed{\theta_T = 76.61729385}$
$\delta_{ST} = 142.9834301$
$\delta_{TT} = 153.2345877$

$\frac{I_S}{s} = 2.285227017$
$\frac{I_T}{s} = 2.42671160$
$\frac{M}{s} = 2.494446335$
$\frac{O}{s} = 2.687426747$



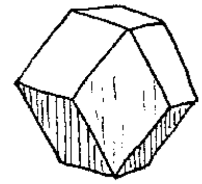
RHOMBIC DODECAHEDRON

$\frac{I}{s_1} = 1.632993162$
$\frac{M}{s_1} = 1.885618083$
$\frac{O_3}{s_1} = 2$
$\frac{O_4}{s_1} = 2.309401077$

$\boxed{\theta = 60}$
$\delta = 120$

$\xi_1 = 109.4712206$
$\xi_2 = 70.52877937$

12 RHOMBI
14 VERTICES
24 EDGES



HEXAKIS OCTAHEDRON

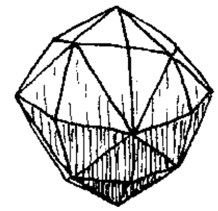
$\frac{I}{s_1} = 1.869135702$
$\frac{M}{s_1} = 1.914213562$
$\frac{N_4}{s_1} = 1.96271850$
$\frac{N_6}{s_1} = 2.07192983$
$\frac{N_8}{s_1} = 2.263033439$
$\frac{s_1}{s_2} = 1.21895142$
$\frac{s_1}{s_3} = 1.630601938$

$\boxed{\theta = 77.54108981}$
$\delta = 155.0821796$

24 LIKE THIS
24 LIKE THIS

$\xi_1 = 87.20196377$
$\xi_2 = 55.02469615$
$\xi_3 = 37.77334008$

48 SCALENE TRIANGLES
26 VERTICES
72 EDGES



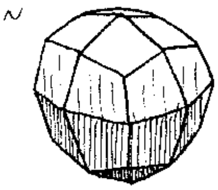
TRAPEZOIDAL ICOSITETRAHEDRON

$\frac{I}{s_1} = 2.254751935$
$\frac{M}{s_1} = 2.414213562$
$\frac{N_3}{s_1} = 2.47538817$
$\frac{N_4}{s_1} = 2.61312593$
$\frac{s_1}{s_2} = 1.29289322$

$\boxed{\theta = 69.05897953}$
$\delta = 138.1179591$

$\xi_1 = 115.2631744$
$\xi_2 = 81.57894188$

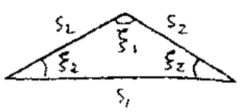
24 KITES
26 VERTICES
48 EDGES



TRIAKIS OCTAHEDRON

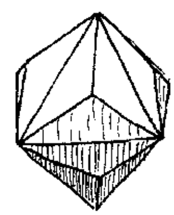
$\frac{I}{s_1} = .959682982$
$\frac{M}{s_1} = 1$
$\frac{O_3}{s_1} = 1.01461187$
$\frac{O_8}{s_1} = 1.414213562$
$\frac{s_1}{s_2} = 1.707106781$

$\boxed{\theta = 73.67505006}$
$\delta = 147.3501001$



$\xi_1 = 117.2005704$
$\xi_2 = 31.39971481$

24 ISOSCELES TRIANGLES
14 VERTICES
36 EDGES

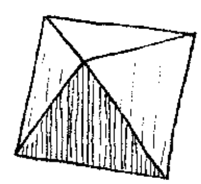


OCTAHEDRON

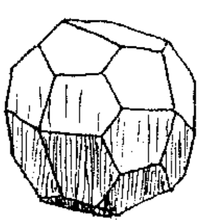
$\frac{I}{s} = .8164965809$
$\frac{M}{s} = 1$
$\frac{O}{s} = 1.414213562$

$\boxed{\theta = 54.73561032}$
$\delta = 109.4712206$

8 TRIANGLES
6 VERTICES
12 EDGES

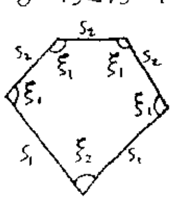


PENTAGONAL ICOSITETRAHEDRON



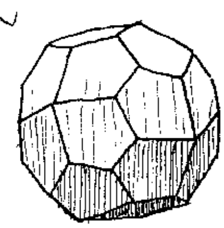
$\frac{I}{s_1} = 2.74812867$
$\frac{M}{s_1} = 2.96073496$
$\frac{O_3}{s_1} = 3.04337544$
$\frac{O_4}{s_1} = 3.23179904$
$\frac{s_1}{s_2} = 1.41964338$

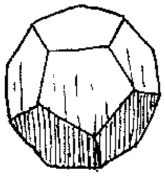
$\boxed{\theta = 68.1546164}$
$\delta = 136.309233$



$\xi_1 = 114.8120745$
$\xi_2 = 80.75170208$

24 IRREGULAR PENTAGONS
38 VERTICES
60 EDGES





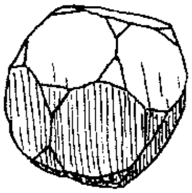
DODECAHEDRON

12 PENTAGONS
20 VERTICES
30 EDGES

$\boxed{\theta_P = 58.28252559}$

$\delta_{PP} = 116.5650512$

$\frac{I}{S} = 2.227032729$
$\frac{M}{S} = 2.618033989$
$\frac{O}{S} = 2.802517077$



TRUNCATED DODECAHEDRON

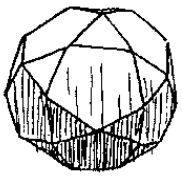
12 DECAGONS (10 SIDES)
20 TRIANGLES

60 VERTICES
90 EDGES

$\boxed{\theta_D = 58.28252559}$
$\boxed{\theta_T = 84.34010627}$

$\delta_{DD} = 116.5650512$
$\delta_{DT} = 142.6226319$

$\frac{I_D}{S} = 4.97979657$
$\frac{I_T}{S} = 5.852556233$
$\frac{M}{S} = 5.854101966$
$\frac{O}{S} = 5.938898032$



ICOSIDODECAHEDRON

12 PENTAGONS
20 TRIANGLES

30 VERTICES
60 EDGES

$\boxed{\theta_P = 63.43494882}$
$\boxed{\theta_T = 79.18768304}$

$\delta_{PT} = 142.6226319$

$\frac{I_P}{S} = 2.752763841$
$\frac{I_T}{S} = 3.023045256$
$\frac{M}{S} = 3.077683537$
$\frac{O}{S} = 3.236067978$

TRUNCATED ICOSIDODECAHEDRON

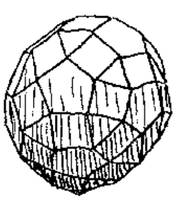
12 DECAGONS
20 HEXAGONS
30 TRIANGLES

120 VERTICES
180 EDGES

$\boxed{\theta_D = 65.40515745}$
$\boxed{\theta_H = 76.71747441}$
$\boxed{\theta_S = 82.37736814}$

$\delta_{DH} = 142.6226319$
$\delta_{DS} = 148.2825256$
$\delta_{HS} = 159.0948426$

$\frac{I_D}{S} = 6.881909602$
$\frac{I_H}{S} = 7.337084961$
$\frac{I_S}{S} = 7.472135955$
$\frac{M}{S} = 7.538754256$
$\frac{O}{S} = 7.60478900$



RHOMBICOSIDODECAHEDRON

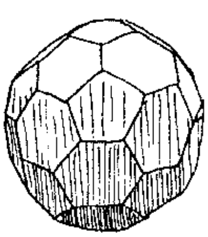
12 PENTAGONS
30 SQUARES
20 TRIANGLES

60 VERTICES
120 EDGES

$\boxed{\theta_P = 71.56505118}$
$\boxed{\theta_S = 76.71747441}$
$\boxed{\theta_T = 82.37736814}$

$\delta_{PS} = 148.28252559$
$\delta_{ST} = 159.0948426$

$\frac{I_P}{S} = 4.129145761$
$\frac{I_S}{S} = 4.236067977$
$\frac{I_T}{S} = 4.314039705$
$\frac{M}{S} = 4.352501799$
$\frac{O}{S} = 4.465901019$



TRUNCATED ICOSAHEDRON

20 HEXAGONS
12 PENTAGONS

60 VERTICES
90 EDGES

$\boxed{\theta_H = 69.09484256}$
$\boxed{\theta_P = 73.52778932}$

$\delta_{HH} = 138.1896851$
$\delta_{HP} = 142.6226319$

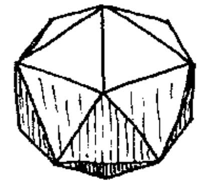
$\frac{I_H}{S} = 4.534567884$
$\frac{I_P}{S} = 4.654876874$
$\frac{M}{S} = 4.854101966$
$\frac{O}{S} = 4.956037318$

$\frac{I}{S} = 1.511522628$
$\frac{M}{S} = 1.618033989$
$\frac{O}{S} = 1.902113033$

$\boxed{\theta_T = 69.09484255}$
$\delta_{TT} = 138.1896851$

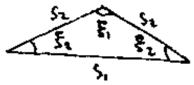
ICOSAHEDRON

20 TRIANGLES
12 VERTICES
30 EDGES



$\frac{I}{S_1} = 1.59493157$
$\frac{M}{S_1} = 1.61803399$
$\frac{O_3}{S_1} = 1.62596079$
$\frac{O_{10}}{S_1} = 1.90211303$
$\frac{S_1}{S_2} = 1.7236068$

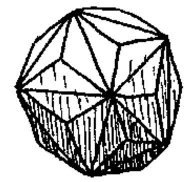
$\boxed{\theta = 80.3062761}$
$\delta = 160.6125522$



$\xi_1 = 119.0393509$
$\xi_2 = 30.48032457$

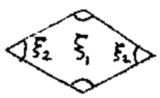
TRIAKIS ICOSAHEDRON

60 ISOSCELES TRIANGLES
32 VERTICES
90 EDGES



$\frac{I}{S_1} = 2.752763841$
$\frac{M}{S_1} = 2.894427191$
$\frac{O_3}{S_1} = 2.94674084$
$\frac{O_5}{S_1} = 3.236067978$

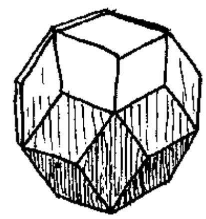
$\boxed{\theta = 72}$
$\delta = 144$



$\xi_1 = 116.5650512$
$\xi_2 = 63.43494882$

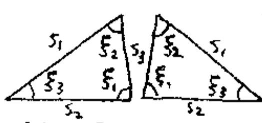
RHOMBIC TRIACONTAHEDRON

30 RHOMBI
32 VERTICES
60 EDGES



$\frac{I}{S_1} = 2.86187618$
$\frac{M}{S_1} = 2.92705098$
$\frac{O_4}{S_1} = 2.9531473$
$\frac{O_6}{S_1} = 3.00750478$
$\frac{O_{10}}{S_1} = 3.206423701$
$\frac{S_1}{S_2} = 1.17595468$
$\frac{S_1}{S_3} = 1.84721360$

$\boxed{\theta = 82.44394595}$
$\delta = 164.8878919$

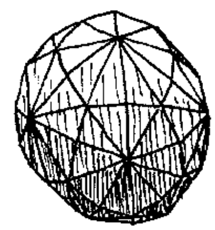


60 LIKE THIS   60 LIKE THIS

$\xi_1 = 88.99180191$
$\xi_2 = 58.23791962$
$\xi_3 = 32.77027847$

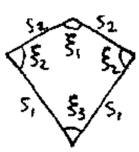
HEXAKIS ICOSAHEDRON

120 SCALENE TRIANGLES
62 VERTICES
180 EDGES



$\frac{I}{S_1} = 3.42327193$
$\frac{M}{S_1} = 3.51246118$
$\frac{O_3}{S_1} = 3.5437768$
$\frac{O_4}{S_1} = 3.6090057$
$\frac{O_5}{S_1} = 3.70245917$
$\frac{S_1}{S_2} = 1.53934466$

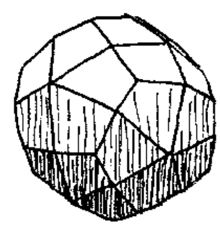
$\boxed{\theta = 77.06068156}$
$\delta = 154.1213631$



$\xi_1 = 118.2686775$
$\xi_2 = 86.97415549$
$\xi_3 = 67.78301155$

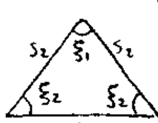
TRAPEZOIDAL HEXECONTAHEDRON

60 KITES
62 VERTICES
120 EDGES



$\frac{I}{S_1} = 2.56418649$
$\frac{M}{S_1} = 2.61803399$
$\frac{O_5}{S_1} = 2.730083798$
$\frac{O_6}{S_1} = 2.80251708$
$\frac{S_1}{S_2} = 1.127322004$

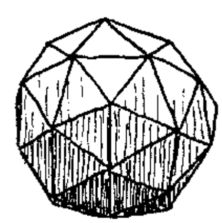
$\boxed{\theta = 78.35927686}$
$\delta = 156.7185537$

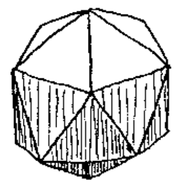


$\xi_1 = 68.61872093$
$\xi_2 = 55.69063953$

PENTAKIS DODECAHEDRON

60 ISOSCELES TRIANGLES
32 VERTICES
90 EDGES





ICOSAHEDRON

20 TRIANGLES

12 VERTICES

30 EDGES

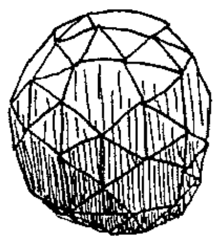
$\boxed{\theta_T = 69.09484255}$

$\delta_{TT} = 138.1896851$

$\frac{I}{s} = 1.511522628$

$\frac{M}{s} = 1.618033989$

$\frac{O}{s} = 1.902113033$



SNUB DODECAHEDRON

12 PENTAGONS

80 TRIANGLES

60 VERTICES

150 EDGES

$\boxed{\theta_P = 70.8422373}$

$\boxed{\theta_T = 82.0876830}$

$\delta_{PT} = 152.9299203$

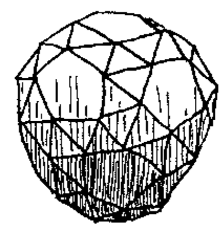
$\delta_{TT} = 164.175366$

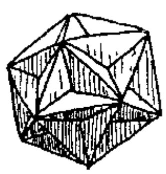
$\frac{I_P}{s} = 3.96183189$

$\frac{I_T}{s} = 4.15417932$

$\frac{M}{s} = 4.19410767$

$\frac{O}{s} = 4.48905703$





GREAT DODECAHEDRON

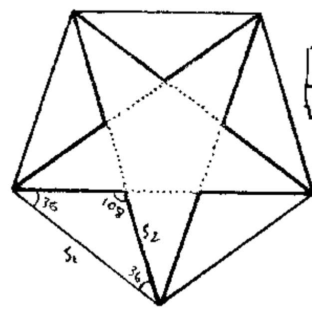
12 INTERSECTING PENTAGONS

12 OUTER VERTICES

20 INTERSECTION VERTICES

30 OUTER EDGES

60 INTERSECTION EDGES



$\boxed{\theta_1 = 31.7174744}$

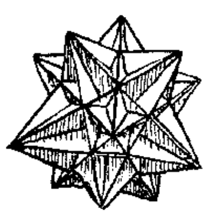
$\boxed{\bar{\theta}_2 = 58.2825256}$

$\delta_1 = 63.4349488$

$\bar{\delta}_2 = 116.5650512$

$\frac{s_1}{s_2} = 1.618033989$

$\frac{O}{s_1} = 1.902113033$



GREAT ICOSAHEDRON

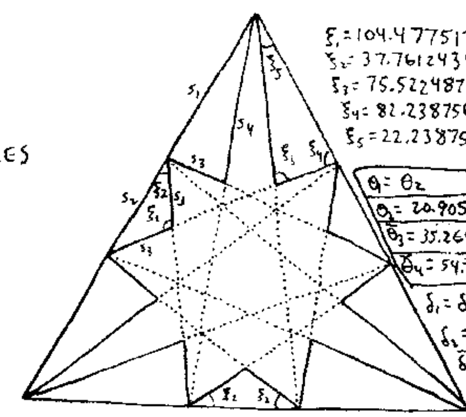
20 INTERSECTING TRIANGLES

12 OUTER VERTICES

72 OTHER VERTICES

30 TRIANGLE EDGES

180 OTHER EDGES



$\xi_1 = 104.4775121$

$\xi_2 = 37.76124391$

$\xi_3 = 75.5224878$

$\xi_4 = 82.23875609$

$\xi_5 = 22.23875609$

$\boxed{\theta_1 = \theta_2}$

$\boxed{\theta_2 = 20.9051575}$

$\boxed{\bar{\theta}_3 = 35.2643897}$

$\boxed{\bar{\theta}_4 = 54.7356103}$

$\delta_1 = \delta_2$

$\delta_2 = 41.8103149$

$\bar{\delta}_3 = 70.5287794$

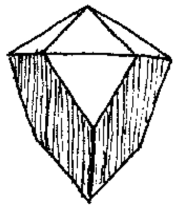
$\bar{\delta}_4 = 109.4712206$

$\frac{s_1}{s_2} = 1.618033989$

$\frac{s_2}{s_3} = 1.58113883$

$\frac{s_3}{s_4} = .381966011$

$\frac{O}{s_1} = 3.30604687$



TRIDIMINISHED ICOSAHEDRON

3 PENTAGONS

5 TRIANGLES

9 VERTICES

15 EDGES

$\boxed{\theta_P = 31.71747441}$

$\boxed{\theta_T = 69.09484255}$

$\delta_{PP} = 63.43494882$

$\delta_{PT} = 100.8123170$

$\delta_{TT} = 138.1896851$

$\frac{I}{s} = .8506508084$

$\frac{M}{s} = 1.618033989$

$\frac{O}{s} = 1.902113033$

$\frac{I}{s} = 2.227032729$

$\frac{M}{s} = 2.618033989$

$\frac{O}{s} = 2.802517077$

$\boxed{\theta = 58.28252559}$

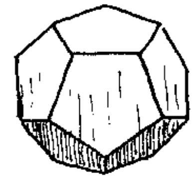
$\delta = 116.5650512$

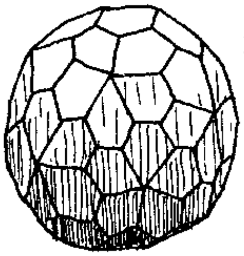
DODECAHEDRON

12 PENTAGONS

20 VERTICES

30 EDGES





$\frac{I}{S_1} = 4.0777463$

$\frac{M}{S_1} = 4.1119176$

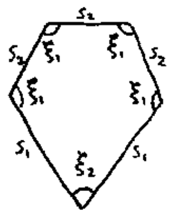
$\frac{O_3}{S_1} = 4.1514398$

$\frac{O_5}{S_1} = 4.3529927$

$\frac{S_1}{S_2} = 1.7498526$

$\boxed{\theta = 76.5893663}$

$\delta = 153.1787326$



$\xi_1 = 118.1366228$

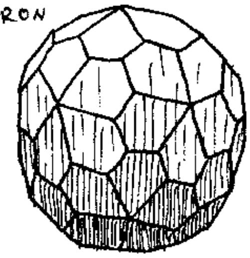
$\xi_2 = 67.4535089$

PENTAGONAL HEXECONTAHEDRON

60 IRREGULAR PENTAGONS

92 VERTICES

150 EDGES

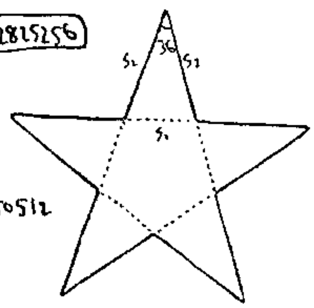


$\boxed{\theta_1 = \theta_2 = 58.2825256}$

$\frac{S_1}{S_2} = .6180339887$

$\frac{O}{S_1} = 5.349296196$

$\delta_1 = \delta_2 = 116.5650512$



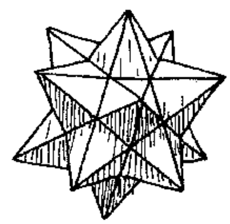
SMALL STELLATED DODECAHEDRON

12 INTERSECTING PENTAGRAMS

12 OUTER VERTICES

20 INTERSECTION VERTICES

30 PENTAGRAM EDGES

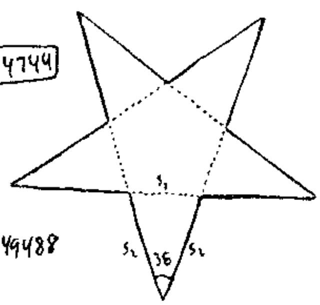


$\boxed{\theta_1 = \theta_2 = 31.7174744}$

$\frac{S_1}{S_2} = .6180339887$

$\frac{O}{S_1} = 4.695671278$

$\delta_1 = \delta_2 = 63.4349488$



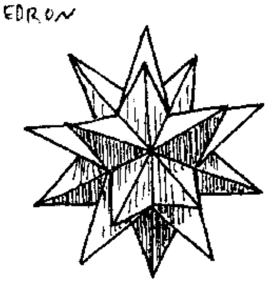
GREAT STELLATED DODECAHEDRON

12 INTERSECTING PENTAGRAMS

20 OUTER VERTICES

12 INTERSECTION VERTICES

30 PENTAGRAM EDGES



$\frac{I}{S_1} = .8506508084$

$\frac{M}{S_1} = 1$

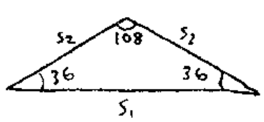
$\frac{O}{S_1} = 2.752763841$

$\frac{S_1}{S_2} = 1.618033989$

$\frac{S_2}{S_3} = 2.618033989$

$\boxed{\theta = 58.28252559}$

$\delta = 116.5650512$



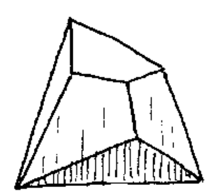
ENNEAHEDRON

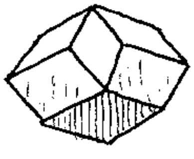
3 KITES

6 ISOSCELES TRIANGLES

8 VERTICES

15 EDGES



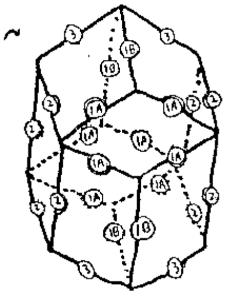


GOLDEN RHOMBIC DODECAHEDRON

12 GOLDEN RHOMBI

14 VERTICES

24 EDGES



$\delta_1 = 144$

$\delta_2 = 108$

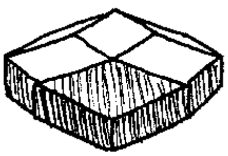
$\delta_3 = 72$

$\delta_{1A} \rightarrow 49.6138224$, $\delta_{1A} \rightarrow 94.3861776$

$\delta_{1B} \xrightarrow{1/2} 72$

$\delta_2 \rightarrow 62.2676986$, $\delta_2 \rightarrow 45.7323015$

$\delta_3 \xrightarrow{1/2} 36$

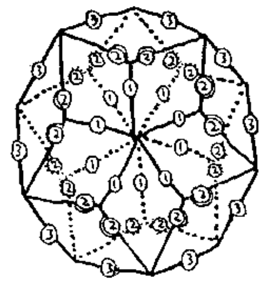


GOLDEN RHOMBIC ICOSAHEDRON

20 GOLDEN RHOMBI

22 VERTICES

40 EDGES



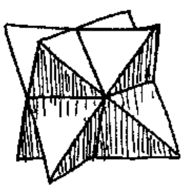
$\delta_1 = \delta_2 = 144$

$\delta_3 = 108$

$\delta_1 \xrightarrow{1/2} 72$

$\delta_2 \rightarrow 58.3861776$, $\delta_2 \rightarrow 85.6138224$

$\delta_3 \xrightarrow{1/2} 54$



STELLA OCTANGULA

24 TRIANGLES

14 VERTICES

36 EDGES

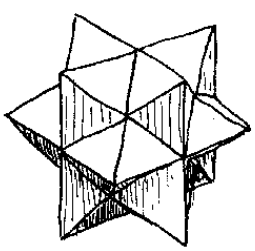
$\boxed{\theta = 35.26438968}$

$\delta = 70.52877937$

$\bar{\delta} = 109.4712206$

$\boxed{\bar{\theta} = 54.7356103}$

$\frac{O}{S} = 3.464101615$

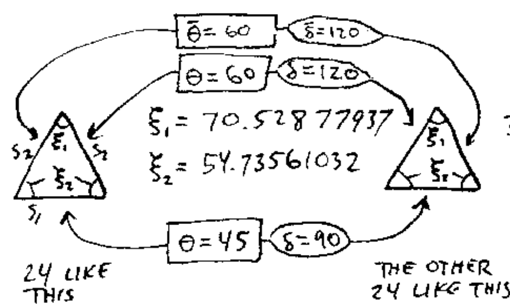


STELLATED RHOMBIC DODECAHEDRON

48 ISOSCELES TRIANGLES

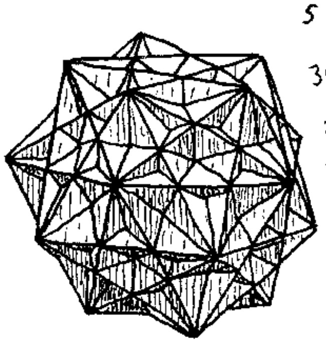
26 VERTICES

72 EDGES



$\frac{S_1}{S_2} = 1.154700538$

$\frac{O}{S_1} = 2.828427125$

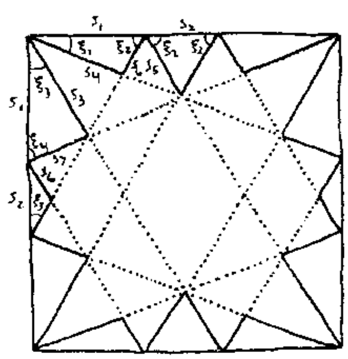


5 CUBES

360 VARIOUS TRIANGLES

20 CUBE VERTICES

60 CUBE EDGES



$\xi_1 = 90 - \xi_4 = 20.9051574$

$\xi_2 = 58.2825256$

$\xi_3 = 90 - \xi_2 = 31.7174744$

$\xi_4 = 69.0948426$

$\delta_1 = \delta_2 = 90$

$\bar{\delta}_3 = \bar{\delta}_5 = 144$

$\bar{\delta}_4 = \bar{\delta}_7 = 120$

$\bar{\delta}_6 = 108$

$\frac{S_1}{S_2} = 1.61803399$

$\frac{S_2}{S_3} = .5877853$

$\frac{S_3}{S_4} = 1.2141240$

$\frac{S_4}{S_5} = 1.4733704$

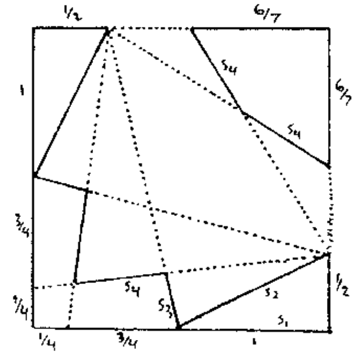
$\frac{S_5}{S_6} = 1.61803399$

$\frac{S_6}{S_7} = .6454972$

$\frac{s_2}{s_1}=1.1180340$ $\quad \delta_1=90$

$\frac{s_3}{s_1}=.3748278$ $\quad \bar{\delta}_2=\bar{\delta}_4=116.387800$

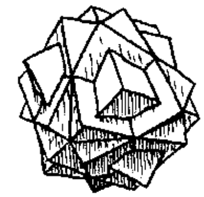
$\frac{s_4}{s_1}=.6282279$ $\quad \bar{\delta}_3=152.733956$

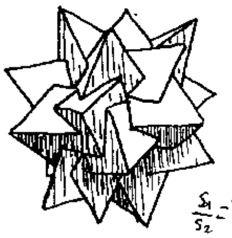


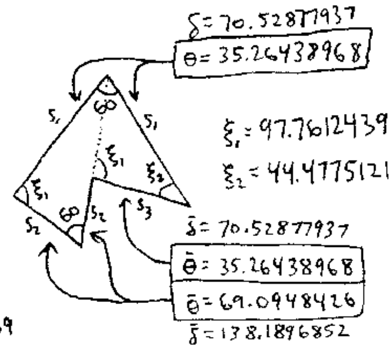
FOUR CUBES

24 L-SHAPED HEXAGONS
24 QUADRILATERALS
48 SCALENE TRIANGLES

32 CUBE VERTICES
48 CUBE EDGES







$\xi_1=97.76124391$

$\xi_2=44.47751219$

$\frac{s_1}{s_2}=2.288245611$

$\frac{s_1}{s_3}=1.618033989$

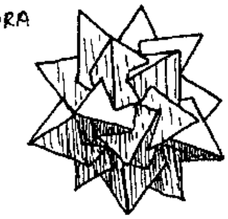
$\frac{O}{s_1}=5.374005337$

FIVE INTERSECTING TETRAHEDRA

60 FUNKY PENTAGONS

92 VERTICES

150 EDGES



$\frac{s_1}{s_2}=1.414213562$ $\quad \delta_1=70.52877937$

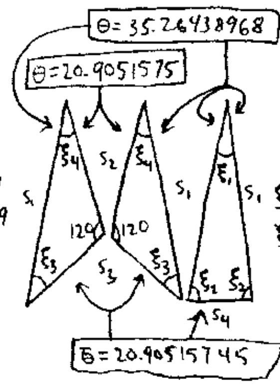
$\delta_2=41.8103149$

$\frac{s_1}{s_3}=2.288245612$

$\frac{s_1}{s_4}=3.702459175$ $\quad \xi_3=37.76124391$, $\xi_4=22.23875609$

$\frac{O}{s_1}=2.654257895$ $\quad \bar{\delta}_3=41.8103149$

$\delta_4=\delta_3$



$\xi_1=15.52248781$

$\xi_2=82.23875609$

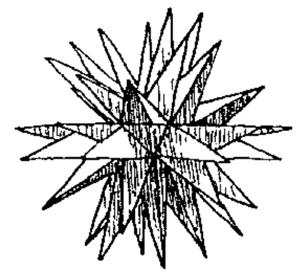
OUTERMOST ICOSAHEDRON STELLATION

120 SCALENE TRIANGLES

60 ISOSCELES TRIANGLES

92 VERTICES

270 EDGES



$\frac{s_1}{s_2}=2.618033989$

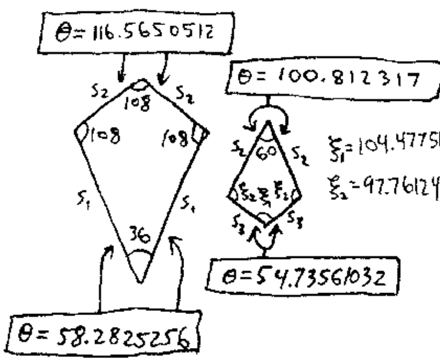
$\frac{s_2}{s_3}=1.58113883$

$\frac{O}{s_1}=4.08649866$

$\delta_1=116.5650512$

$\bar{\delta}_2=142.6226318$

$\delta_3=109.4712206$



$\xi_1=104.4775122$

$\xi_2=97.76124191$

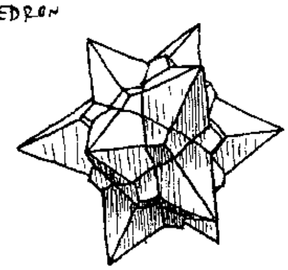
STELLATED ICOSIDODECAHEDRON

60 BIG KITES

60 SMALL KITES

122 VERTICES

240 EDGES



$\frac{s_1}{s_2}=2.618033989$

$\frac{s_1}{s_3}=1.618033989$

$\frac{s_2}{s_4}=1$

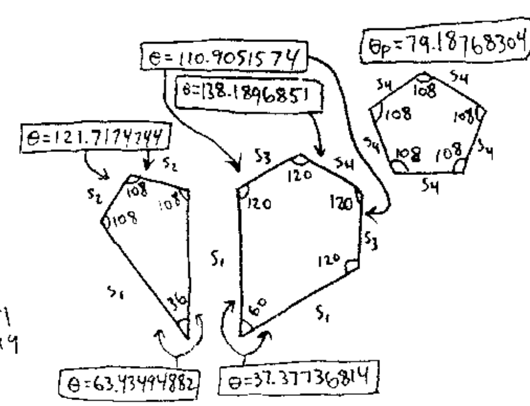
$\frac{O}{s_1}=5.236067978$

$\delta_1=100.8123170$

$\bar{\delta}_2=116.5650512$

$\bar{\delta}_3=138.1896851$

$\bar{\delta}_4=142.6226319$



GREAT ICOSIDODECAHEDRON

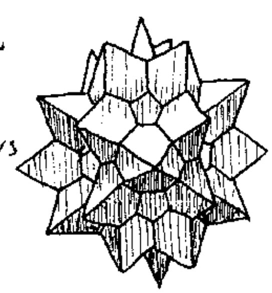
12 PENTAGONS

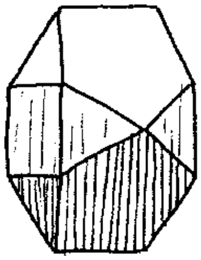
60 IRREGULAR PENTAGONS

60 KITES

170 VERTICES

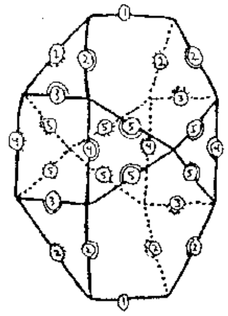
300 EDGES





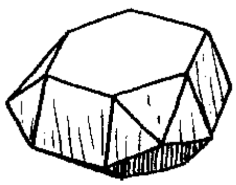
BILUNABIROTUNDA

4 PENTAGONS
2 SQUARES
8 TRIANGLES
14 VERTICES
26 EDGES



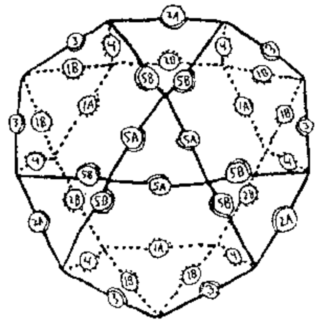
$\delta_1 = 63.4349488$
$\delta_2 = 100.812317$
$\delta_3 = 159.0948426$
$\delta_4 = 110.9051574$
$\delta_5 = 142.6226319$

$\delta_1 \xrightarrow{1/2} 31.7174744$
$\delta_2 \to 40.0182492$, 60.7940678
$\delta_3 \to 100.812317$, 58.2825256
$\delta_4 \to 52.6226318$
$\delta_5 \to 63.4349488$, 79.1876831



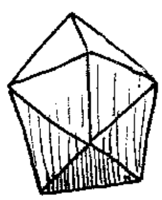
TRIANGULAR HEBESPHENOROTUNDA

1 HEXAGON
3 PENTAGONS
3 SQUARES
13 TRIANGLES
18 VERTICES
36 EDGES



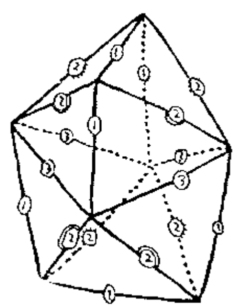
$\delta_1 = 138.1896851$
$\delta_2 = 110.9051574$
$\delta_3 = 100.812317$
$\delta_4 = 159.0948426$
$\delta_5 = 142.6226319$

$\delta_{1B} \to 74.8244669$, 63.3652182
$\delta_{1A} \to 100.90871$, 37.2809751
$\delta_{2B} \to 73.6241823$
$\delta_{2A} \to 56.4255447$, 54.4796129
$\delta_3 \to 40.1104839$, 60.7018331
$\delta_4 \to 64.4145881$, 94.6802544
$\delta_{5B} \to 56.487408$, 86.1352238
$\delta_{5A} \to 71.3351169$, 71.287515



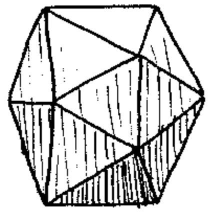
SNUB DISPHENOID

12 TRIANGLES
8 VERTICES
18 EDGES



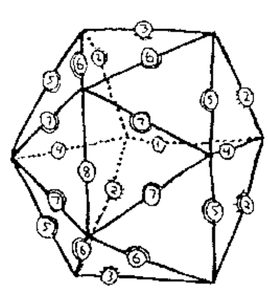
$\delta_1 = 96.1983289$
$\delta_2 = 121.7431573$
$\delta_3 = 166.4405523$

$\delta_1 \xrightarrow{1/2} 48.0991645$
$\delta_2 \to 73.6439928$, 48.0991645
$\delta_3 \xrightarrow{1/2} 83.2202762$



SPHENOCORONA

2 SQUARES
12 TRIANGLES
10 VERTICES
22 EDGES

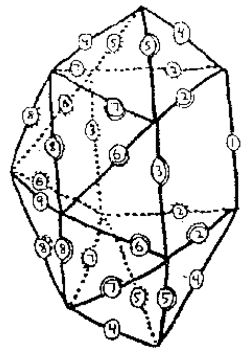


$\delta_1 = 117.019034$
$\delta_2 = 109.524034$
$\delta_3 = 97.455522$
$\delta_4 = 159.892396$
$\delta_5 = 118.892201$
$\delta_6 = 135.991508$
$\delta_7 = 143.478724$
$\delta_8 = 131.441571$

$\delta_1 \xrightarrow{1/2} 58.509517$
$\delta_2 \to 60.978136$, 48.545897
$\delta_3 \to 40.9100142$, 56.545511
$\delta_4 \xrightarrow{1/2} 79.946198$
$\delta_5 \to 59.920431$, 58.971772
$\delta_6 \to 74.887481$, 61.104027
$\delta_7 \to 80.312014$, 63.166709
$\delta_8 \xrightarrow{1/2} 65.7207857$

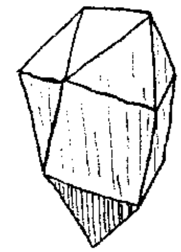
36.4864978 $\xleftarrow{1/2}$ δ_1
47.4665918
107.2556921 δ_2
64.2324599
73.0076239 δ_3
43.3634151 $\xleftarrow{1/2}$ δ_4
64.4578024
64.9867653 δ_5
59.2319443
84.5063819 δ_6
106.9438753
64.7018607 δ_7
54.1406609
63.2149042 δ_8
80.7414247 $\xleftarrow{1/2}$ δ_9

$\delta_1 = 72.972996$
$\delta_2 = 154.722284$
$\delta_3 = 137.240084$
$\delta_4 = 86.726830$
$\delta_5 = 129.444568$
$\delta_6 = 143.738326$
$\delta_7 = 171.645736$
$\delta_8 = 117.355565$
$\delta_9 = 161.482849$



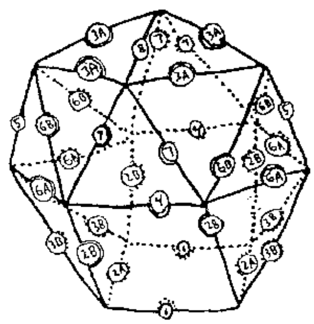
SPHENOMEGACORONA

2 SQUARES
16 TRIANGLES
12 VERTICES
28 EDGES



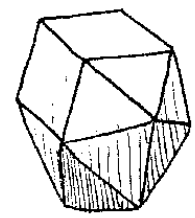
43.1171683
59.4066128 δ_1
74.5661922 δ_{2A}
53.3279007
80.6449094 δ_{2B}
71.8866892
56.609292 δ_{3A}
72.2966414
56.1993407 δ_{3B}
67.1563561
85.8192224 δ_4
55.8673883 $\xleftarrow{1/2}$ δ_5
77.9109099
79.2372382 δ_{6A}
77.1034513
80.0446968 δ_{6B}
60.7889125
80.5601921 δ_7
74.7824153 $\xleftarrow{1/2}$ δ_8

$\delta_1 = 102.523781$
$\delta_2 = 133.972805$
$\delta_3 = 128.495982$
$\delta_4 = 152.975579$
$\delta_5 = 111.734777$
$\delta_6 = 157.148148$
$\delta_7 = 141.341105$
$\delta_8 = 149.564831$



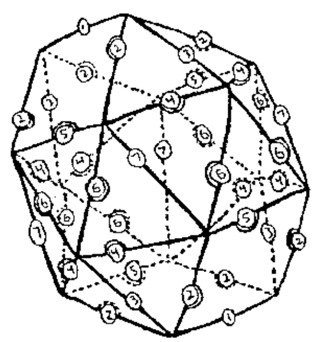
HEBESPHENOMEGACORONA

3 SQUARES
18 TRIANGLES
14 VERTICES
33 EDGES



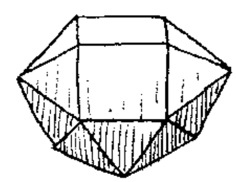
50.0969598 $\xleftarrow{1/2}$ δ_1
59.4532123
76.8827321 δ_2
62.3509594 $\xleftarrow{1/2}$ δ_3
81.6383987
66.7955937 δ_4
66.7955937 $\xleftarrow{1/2}$ δ_6
71.01314423
83.4056851 δ_5
83.4056851 $\xleftarrow{1/2}$ δ_7

$\delta_1 = 100.193920$
$\delta_2 = 136.335944$
$\delta_3 = 124.701919$
$\delta_4 = 126.347812$
$\delta_5 = 154.418829$
$\delta_6 = 133.591187$
$\delta_7 = 166.811370$



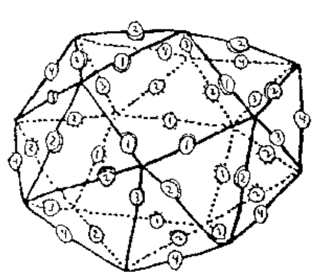
DISPHENOCINGULUM

4 SQUARES
20 TRIANGLES
16 VERTICES
38 EDGES



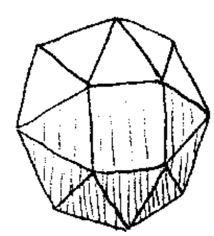
91.8937599
53.5468694 δ_1
82.1286975
62.0149231 δ_2
82.1286975 $\xleftarrow{1/2}$ δ_3
57.322604 $\xleftarrow{1/4}$ δ_4

$\delta_1 = 145.440629$
$\delta_2 = 144.143621$
$\delta_3 = 164.257395$
$\delta_4 = 114.645209$



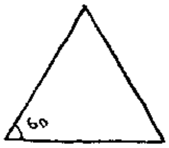
SNUB SQUARE ANTIPRISM

2 SQUARES
24 TRIANGLES
16 VERTICES
40 EDGES

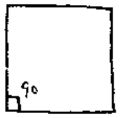


MATH STUFF

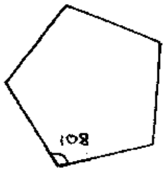
OPTIONAL ON FIRST READING



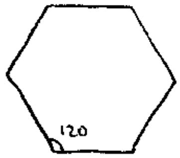
TRIANGLE



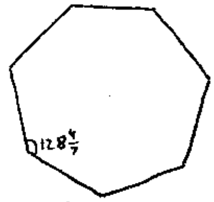
SQUARE



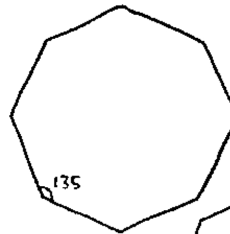
PENTAGON



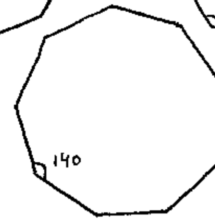
HEXAGON



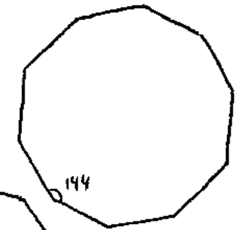
SEPTAGON



OCTAGON

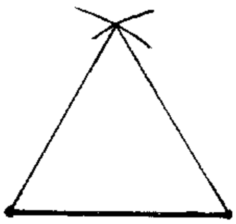


NONAGON



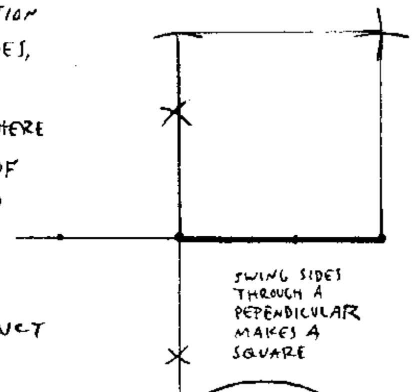
DECAGON

CONSTRUCTIONS

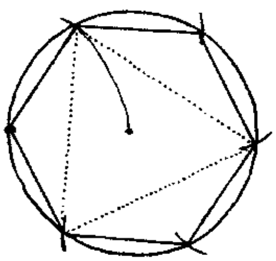


TRIANGLE FROM AN EDGE

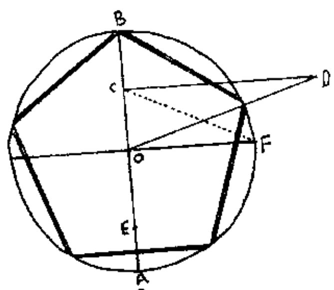
KARL FRIEDRICH GAUSS (1777-1855) SHOWED THAT IN ADDITION TO REGULAR POLYGONS OF 2^n, $3\cdot 2^n$, $5\cdot 2^n$, AND $15\cdot 2^n$ SIDES, ONE MAY ALSO CONSTRUCT WITH A STRAIGHT EDGE AND COMPASS POLYGONS WITH THE NUMBER OF SIDES $2^n\cdot a$ WHERE a IS A PRODUCT OF DIFFERENT PRIME NUMBERS OF THE FORM $1+2^{2^m}$. SOME SUCH m ARE $m=0,1,2,3,4$, AND NO, 5 DOES NOT WORK.



SWING SIDES THROUGH A PEPENDICULAR MAKES A SQUARE

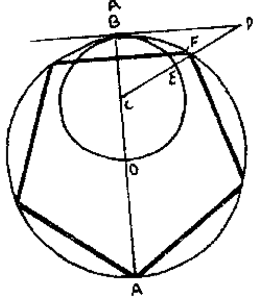


HEXAGON AND TRIANGLE IN A CIRCLE

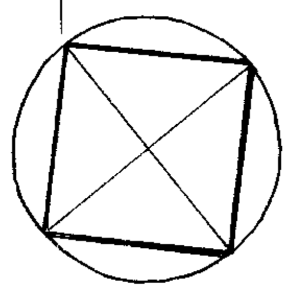


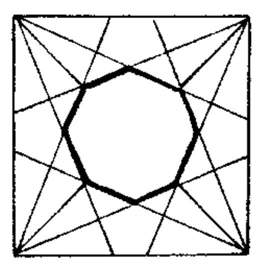
HERE ARE 2 WAYS TO CONSTRUCT THE PENTAGON.

IN CIRCLE O, DRAW OF PERPENDICULAR TO AB. TAKE $OC=\frac{1}{2}OB$. DRAW A LINE AT C PARALLEL TO OF. MAKE CE=CF, AND OD=BE. THEN $\angle BOD = 72$. THIS CONSTRUCTION IS DERIVED FROM THE RELATION $\cos 72 = \frac{1}{2\tau}$ WHERE $\tau^2=\tau+1$.



IN CIRCLE O, DRAW CIRCLE C WITH RADIUS CO. DRAW BD PERPENDICULAR TO AB WITH BD=OB. THEN DE=BF, BF BEING A CHORD OF CIRCLE O. F IS A VERTEX OF THE PENTAGON, AND DOES NOT LIE ON CD.

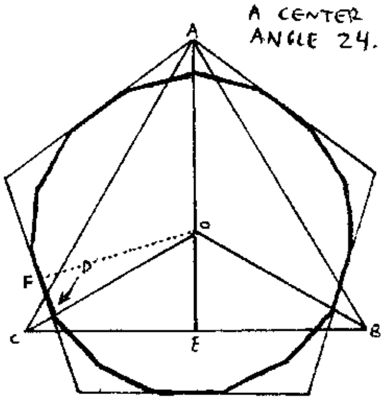




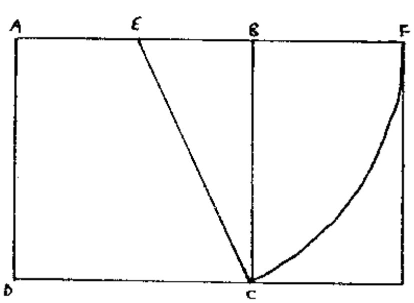
AN OCTAGON MAY BE FOUND IN A SQUARE BY TWICE BISECTING ITS RIGHT ANGLES

FOR MANY CONSTRUCTIONS FROM SQUARES, SEE SUNDARA ROW'S GEOMETRIC EXERCISES IN PAPER FOLDING

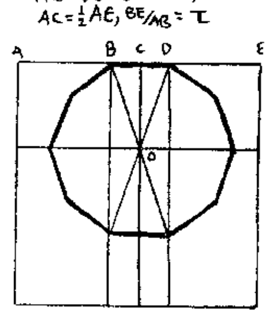
A PENTADECAGON CAN BE INSCRIBED IN A PENTAGON WITH THE AID OF A TRIANGLE. LET O BE THE CENTER. SINCE $\angle AOC - \angle AOF = 120-108=12$, POINT D IS A VERTEX OF THE 15-GON WHICH HAS A CENTER ANGLE 24.



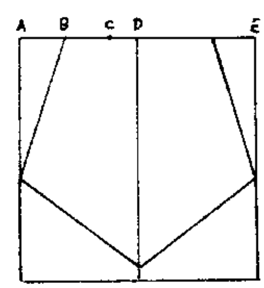
IN SQUARE ABCD, TAKE E THE MIDPOINT OF AB, MAKE EF=EC. THE RESULTING RECTANGLE IS CALLED THE GOLDEN RECTANGLE AND THE RATIO $\frac{AF}{AB}=\frac{AB}{BF}=\tau$ THE GOLDEN RATIO, WHICH WILL BE DEALT WITH LATER IN ITS OWN SECTION.



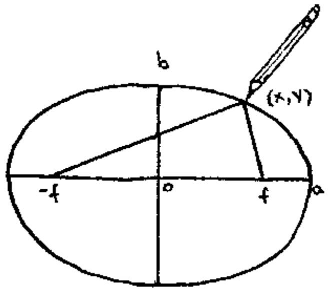
A DECAGON MAY BE FORMED IN A SQUARE BY TAKING AB=DE=OB=OD, $AC=\frac{1}{2}AE$, $BE/AB=\tau$

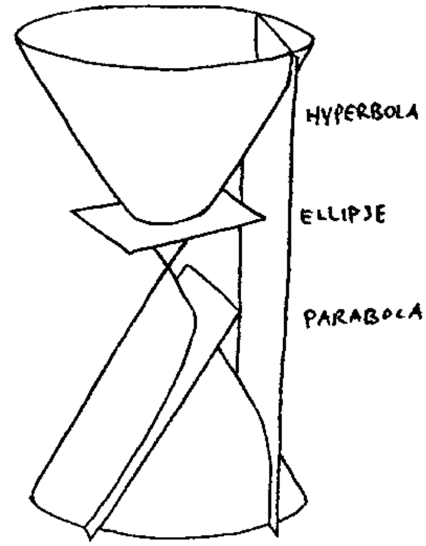


A PENTAGON MAY BE MADE IN A SQUARE BY TAKING $AD=\frac{1}{2}AE$, $AB=\frac{1}{2}BC$, AND $CE/AC=\tau$.



CONIC SECTIONS





ELLIPSE

THE SUM OF DISTANCES OF EACH POINT OF AN ELLIPSE TO THE SO-CALLED FOCI OF THE ELLIPSE IS EQUAL TO A CONSTANT. CALL THIS CONSTANT C.

SO $\sqrt{(x+f)^2+y^2}+\sqrt{(x-f)^2+y^2}=c$

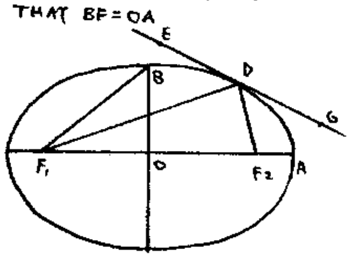
WHICH SIMPLIFIES TO $\frac{4}{c^2}x^2+\frac{4}{c^2-4f^2}y^2=1$.

AT $x,y=0,b$ WE SEE $b^2+f^2=(\frac{c}{2})^2$

AT $x,y=a,0$ WE SEE $f+a+(a-f)=2a=c$

SO THE ELLIPSE IS $\boxed{(\frac{x}{a})^2+(\frac{y}{b})^2=1,\ b^2+F^2=a^2}$

DRAWING AN ELLIPSE CAN BE DONE BY DRAGGING A PENCIL AROUND A FIXED LOOP OF STRING AS IN DIAGRAM. IF YOU KNOW HOW BIG AN ELLIPSE YOU WANT, THE FOCI ARE FOUND BY NOTICING THAT BF=OA



WITH TANGENT EG, THE INDUSTRIOUS STUDENT CAN SHOW THAT $\angle F_1DE=\angle F_2DG$, THUS LIGHT FROM ONE FOCUS WILL BOUNCE OFF A MIRRORED ELLIPSE TO COLLECT AT THE OTHER FOCUS.

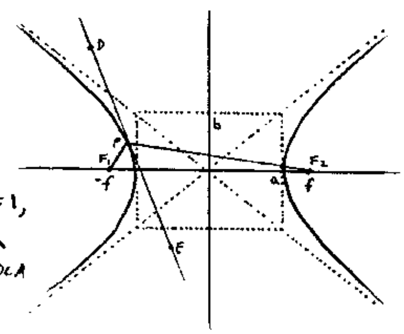
HYPERBOLA

IN CONTRAST TO THE ELLIPSE, THE POINTS OF THE HYPERBOLA HAVE DISTANCES TO TWO FOCI WHOSE DIFFERENCE IS A CONSTANT, SAY C. THUS,

$\sqrt{(x+f)^2+y^2}-\sqrt{(x-f)^2+y^2}=c$

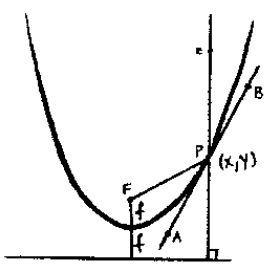
FROM WHICH $\frac{4}{c^2}x^2+\frac{4}{c^2-4f^2}y^2=1$,

LIKE THE ELLIPSE. AGAIN $c=2a$ BUT NOW $f>a$ SO HYPERBOLA IS $\boxed{(\frac{x}{a})^2-(\frac{y}{b})^2=1,\ b^2=f^2-a^2}$



WITH TANGENT DE, ONE MAY SHOW THAT $\angle F_1PE=\angle F_2PE$

PARABOLA

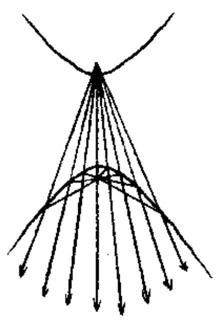


A PARABOLA HAS ITS POINTS EQUIDISTANT FROM A POINT AND A LINE.

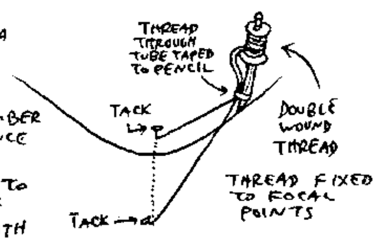
SO $f+y=\sqrt{(y-f)^2+x^2}$

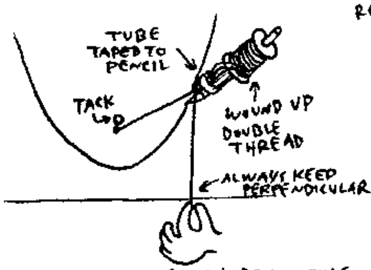
FROM WHICH $\boxed{y=\frac{x^2}{4f}}$

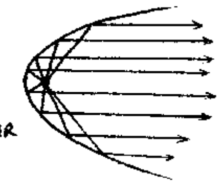
WITH TANGENT AB, ONE MAY SHOW THAT $\angle FPA=\angle CPB$, THUS PERMITTING THE RENOWNED PARABOLIC REFLECTOR



ONE MAY DRAW A HYPERBOLA WITH THE AID OF SUCH A DEVICE AS IN PICTURE. REMEMBER THAT THE DISTANCE BETWEEN FOCAL POINTS IS EQUAL TO THE DIAGONAL OF A RECTANGLE WITH SIDES 2a AND 2b (THE DOTTED RECTANGLE ABOVE)







ONE MAY DRAW THE PARABOLA WITH THE CONTRAPTION IN THE PICTURE. THE DOUBLE WOUND THREAD ENSURES THAT EQUAL LENGTHS OF THREAD WILL UNWIND AT THE SAME TIME. THIS IS BASICALLY IDENTICAL TO DRAWING A HYPERBOLA ONLY THE SECOND FOCUS IS A LINE, AND THE DIFFERENCE IN LENGTHS IS ZERO.

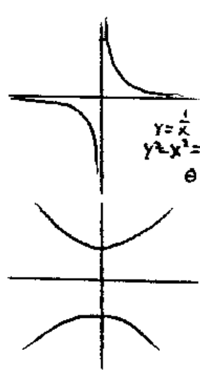
CONICS, THE COORDINATE PLANE, AND YOU

LET US CONSIDER THE GENERAL SECOND ORDER QUADRATIC $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ UNDER THE ROTATION θ DEFINED BELOW LEFT, THIS EQUATION BECOMES

$A_1x_1^2+B_1x_1y_1+C_1y_1^2+D_1x+E_1y+F_1=0$,

$A_1=A\cos^2\theta+C\sin^2\theta+B\cos\theta\sin\theta$
$B_1=B(2\cos^2\theta-1)+2\cos\theta\sin\theta(C-A)$
$C_1=A\sin^2\theta+C\cos^2\theta-B\cos\theta\sin\theta$
$D_1=D\cos\theta+E\sin\theta$
$E_1=E\cos\theta+D\sin\theta$
$F_1=F$

$B_1=0 \Rightarrow \cot 2\theta=\frac{A-C}{B}$

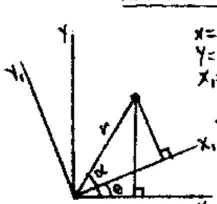


$y=\frac{1}{x}$ BECOMES $y^2-x^2=2$ WHEN $\theta=45$

ONE MAY VERIFY THAT $B^2-4AC=B_1^2-4A_1C_1$, THEREFORE WE MAY DETERMINE THE NATURE OF $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ BY EXAMINING B^2-4AC UNDER THE ROTATION θ, $\cot 2\theta=\frac{A-C}{B}$ WHICH MAKES $B_1=0$. SO

$B^2-4AC<0$, ELLIPSE
$B^2-4AC=0$, PARABOLA
$B^2-4AC>0$, HYPERBOLA

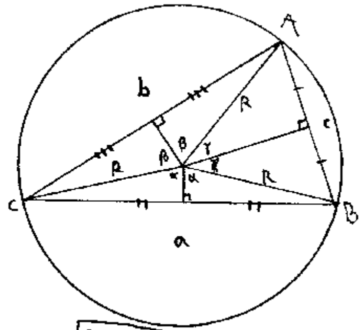
ROTATION OF COORDINATES



$x=r\cos\alpha$
$y=r\sin\alpha$
$x_1=r\cos(\alpha-\theta)=r\cos\alpha\cos\theta+r\sin\alpha\sin\theta$
$=x\cos\theta+y\sin\theta$
$y_1=r\sin(\alpha-\theta)=r\sin\alpha\cos\theta-r\cos\alpha\sin\theta$
$=y\cos\theta-x\sin\theta$

SO ALSO
$x=x_1\cos\theta-y_1\sin\theta$
$y=y_1\cos\theta+x_1\sin\theta$

TRIANGLES



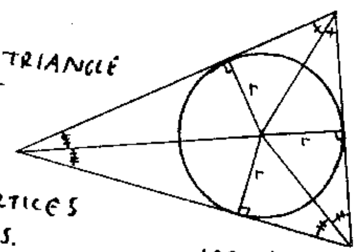
CIRCUMCIRCLE

3 POINTS DETERMINE A CIRCLE SO ANY TRIANGLE MAY HAVE A CIRCLE DRAWN AROUND IT. THIS IS CALLED THE TRIANGLE'S CIRCUMCIRCLE, AND ITS CENTER IS THE CIRCUMCENTER, AT WHICH THE TRIANGLE'S PERPENDICULAR SIDE BISECTORS MEET. FROM THIS, WE MAY DERIVE THE LAW OF SINES:

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}=\frac{1}{2R}$$

AND ALSO AN AREA FORMULA: $\frac{1}{2}ac\sin\beta=\frac{abc}{4R}$

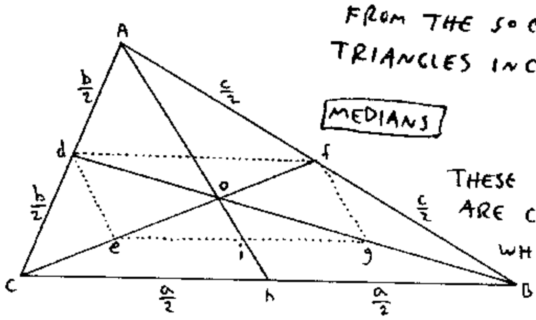
INCIRCLE



ONE MAY ALSO DRAW A CIRCLE WITHIN A TRIANGLE SO THAT ITS EDGES ARE TANGENT TO THE CIRCLE. ONE FINDS FROM THAT THAT LINES FROM THE SO CALLED INCIRCLE'S CENTER (THE TRIANGLES INCENTER) TO THE TRIANGLE'S VERTICES BISECT THE TRIANGLE'S ANGLES.

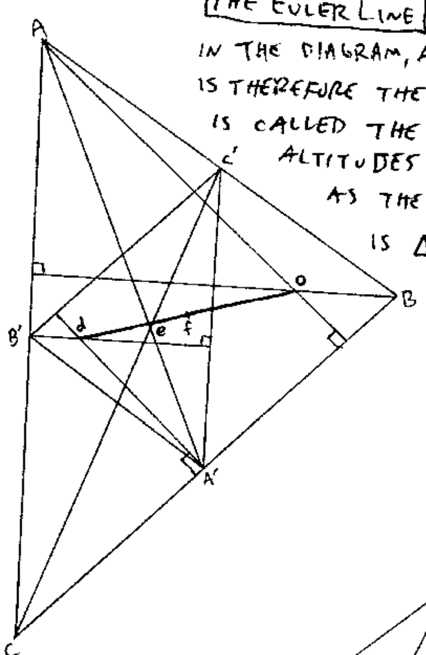
AREA $=\frac{1}{2}ra+\frac{1}{2}rb+\frac{1}{2}rc$
$=r\frac{a+b+c}{2}$

MEDIANS



THESE SIDE BISECTORS (INTERIOR SOLID LINES) ARE CALLED MEDIANS. THEIR COMMON POINT IS THE CENTROID WHICH HERE DIVIDES THE MEDIANS IN RATIO 1:2. SEE THIS AS FOLLOWS. TAKE $de \parallel fg \parallel Ah$. SINCE df IS CUT IN HALF BY Ah, $degf$ IS A PARALLELOGRAM, AND $ei=\frac{1}{2}ch$, SO $oi=\frac{1}{2}oh$ AND $og=\frac{1}{2}oB$ THEREFORE $do=og=gB$, AND SIMILARLY FOR THE OTHER MEDIANS. A CONSEQENCE OF THIS IS THAT THE MEDIANS OF A TRIANGLE DIVIDE IT INTO SIX TRIANGLES OF EQUAL AREA.

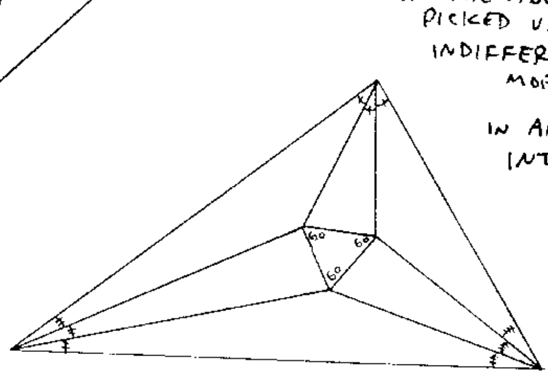
THE EULER LINE



IN THE DIAGRAM, A', B', AND C' DIVIDE THEIR RESPECTIVE SIDES IN HALF. THE POINT e IS THEREFORE THE CENTROID, AND d THE CIRCUMCENTER OF TRIANGLE ABC. $\Delta A'B'C'$ IS CALLED THE MEDIAL TRIANGLE OF ΔABC. POINT o IS THE COMMON POINT OF ALTITUDES OF ΔABC AND IS CALLED THE ORTHOCENTER. POINT d DOUBLES AS THE ORTHOCENTER OF $\Delta A'B'C'$ AND e IS ITS CENTROID ALSO. POINT f IS $\Delta A'B'C'$'S CIRCUMCENTER. ONE MAY SHOW THE MOST AMAZING FACT THAT d, e, f, AND o ARE ALL ON THE SAME LINE, CALLED THE EULER LINE. FURTHERMORE, $df=fo$, $de=\frac{1}{2}eo$.

MORLEY'S THEOREM

IF THE ABOVE DOESN'T IMPRESS YOU, YOU'VE PROBABLY PICKED UP THIS BOOK ACCIDENTALLY. YOU MAY TEST YOUR INDIFFERENCE WITH THE FOLLOWING, KNOWN AS MORLEY'S THEOREM.

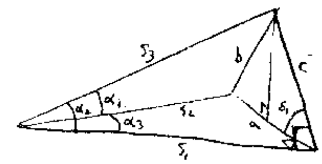


IN ANY TRIANGLE, TRISECT THE ANGLES. THE RESULTING INTERSECTIONS AS IN THE DIAGRAM ALWAYS FORM AN EQUILATERAL TRIANGLE.

FOR A MUCH BETTER TREATMENT OF THESE THINGS, CONSULT GEOMETRY REVISITED BY H.S.M COXETER AND S.L. GREITZER.

CORNERS

GIVEN A CORNER WHERE 3 SIDES MEET, SAY THE ANGLES ON THOSE SIDES ARE α_1, α_2, AND α_3 WHICH SIT OPPOSITE EDGES S_1, S_2, S_3 RESPECTIVELY. ALSO SAY THE DIHEDRAL ANGLES ALONG THOSE SIDES ARE $\delta_1, \delta_2, \delta_3$.



THEN FROM DIAGRAM, $\frac{S_1}{S_2} = \cos\alpha_3$, $\frac{a}{S_2} = \sin\alpha_3 \Rightarrow a = S_1 \tan\alpha_3$

$\frac{S_1}{S_3} = \cos\alpha_2$, $\frac{c}{S_3} = \sin\alpha_2 \Rightarrow c = S_1 \tan\alpha_2$

ALSO $b^2 = S_2^2 + S_3^2 - 2S_2S_3\cos\alpha_1 = a^2 + c^2 - 2ac\cos\delta_1$

SO $\frac{S_1^2}{\cos^2\alpha_3} + \frac{S_1^2}{\cos^2\alpha_2} - 2\frac{S_1^2}{\cos\alpha_3\cos\alpha_2}\cos\alpha_1 = S_1^2\tan^2\alpha_3 + S_1^2\tan^2\alpha_2 - 2S_1^2\tan\alpha_3\tan\alpha_2\cos\delta_1$

WHICH REDUCES TO $\boxed{\cos\delta_1 = \frac{\cos\alpha_1 - \cos\alpha_2\cos\alpha_3}{\sin\alpha_2\sin\alpha_3}}$

IF $\alpha_2 = \alpha_3$ THEN THE FORMULA AT LEFT SIMPLIFIES

$\cos\delta_1 = \frac{\cos\alpha_1 - \cos^2\alpha_2}{\sin^2\alpha_2}$

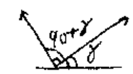
SAY $2\theta = \delta_1$ SO

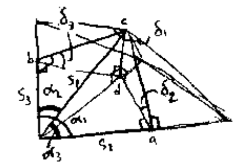
$2\sin^2\theta = \frac{\sin^2\alpha_2 + \cos^2\alpha_2 - \cos\alpha_1}{\sin^2\alpha_2}$

$\boxed{\sin\theta = \frac{\sin\frac{\alpha_1}{2}}{\sin\alpha_2}}$

SUPPOSE YOU HAVE A BASE OF ANGLE α_1 AND YOU WANT SIDES THAT SIT ON THAT BASE WITH CERTAIN INCLINATIONS, SAY δ_2 AND δ_3. ONE WAY TO FIND δ_1 IS BY CONSIDERING VECTORS PERPENDICULAR TO α_3 AND α_2 PLANES.

TAKE $r=1$ IN COORDINATE TRANSFORMATION AT RIGHT.





SO $X_2 = \cos(90+\delta_2)\cos(\alpha_1 - 90)$ $\quad X_3 = \cos(90+\delta_3)\cos 90$

$Y_2 = \cos(90+\delta_3)\sin(\alpha_1 - 90)$ $\quad Y_3 = \cos(90+\delta_2)\sin 90$

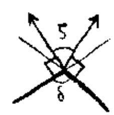
$Z_2 = \sin(90+\delta_3)$ $\quad Z_3 = \sin(90+\delta_2)$

SO $\cos\zeta = (X_2, Y_2, Z_2)\cdot(X_3, Y_3, Z_3) = -\cos\delta_1$

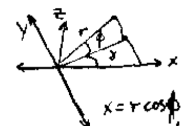
$= \cos\delta_3\cos\delta_2 - \sin\delta_3\sin\delta_2\cos\alpha_1$

SO $\boxed{\cos\delta_1 = \sin\delta_3\sin\delta_2\cos\alpha_1 - \cos\delta_3\cos\delta_2}$

IF $\delta_2 = \delta_3$, THIS SIMPLIFIES TO $\boxed{\cos\theta = \sin\delta_2\cos\frac{\alpha_1}{2}}$ WHERE $2\theta = \delta_1$, WHICH COMBINED WITH THE SIMILAR CASE ABOVE RIGHT GIVES $\boxed{\tan\alpha_2\cos\delta_2 = \tan\frac{\alpha_1}{2}}$



$\delta_1 = 2(90-\zeta) + \zeta = 180 - \zeta$



$x = r\cos\phi\cos\gamma$

$y = r\cos\phi\sin\gamma$

$z = r\sin\phi$

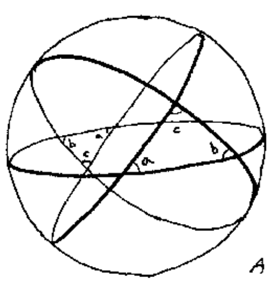
ALSO BY EXAMINING THE DIAGRAM,

$\sin\delta_2 = \frac{cd}{ac}$, $\sin\delta_3 = \frac{cd}{cb}$, $\sin\alpha_3 = \frac{ac}{S_1}$, $\sin\alpha_2 = \frac{cb}{S_1}$

SO $\frac{\sin\delta_2}{\sin\alpha_2} = \frac{\sin\delta_3}{\sin\alpha_3}$. THERE IS NOTHING SPECIAL ABOUT THIS CONFIGURATION OF δ SO:

$\boxed{\frac{\sin\delta_1}{\sin\alpha_1} = \frac{\sin\delta_2}{\sin\alpha_2} = \frac{\sin\delta_3}{\sin\alpha_3}}$

AROUND THE CORNER



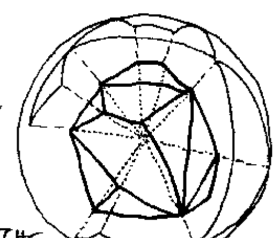
A SPHERICAL TRIANGLE WITH FACE ANGLES a, b, c DIVIDES THE SPHERE INTO 8 REGIONS OF 4 DIFFERENT TYPES, EACH OCCURRING TWICE. CALL THE TRIANGLE'S AREA A. A SPHERE HAS AREA 4π SO

$$2\left(A + \left(2\pi\left(\tfrac{a}{\pi}\right) - A\right) + \left(2\pi\left(\tfrac{b}{\pi}\right) - A\right) + \left(2\pi\left(\tfrac{c}{\pi}\right) - A\right)\right) = 4\pi$$

SO WE FIND THAT $\boxed{A = a + b + c - \pi}$

AND FOR AN N-GON, $\boxed{A = a_1 + a_2 + \dots + a_N - (N-2)\pi}$

SINCE THOSE MAY BE DIVIDED INTO TRIANGLES.



ANY POLYHEDRON THAT CAN BE COMFORTABLY PROJECTED ONTO A SPHERE HAS THE FOLLOWING PROPERTY.

SAY V = NUMBER OF VERTICES, F = # OF FACES, E = # OF EDGES

THE TOTAL AREA ON THE SPHERE IS

4π = (SUM OF AREAS OF ALL PROJECTED POLYGONS)

$= \sum_V \begin{pmatrix}\text{SUM OF SPHERICAL}\\ \text{ANGLES AT}\\ \text{EACH VERTEX}\end{pmatrix} - \sum_F (n_f - 2)\pi$, n_f = # SIDES OF FACE # f

$= \sum_V 2\pi - \sum_V \begin{pmatrix}\text{SUM OF POLYGON}\\ \text{FACE ANGLES AT}\\ \text{EACH VERTEX}\end{pmatrix}$

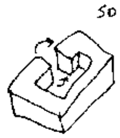
SO $4\pi = \sum_V \left(2\pi - \begin{pmatrix}\text{SUM OF POLY.}\\ \text{FACE ANGLES}\\ \text{AT VERTEX}\end{pmatrix}\right)$

THIS IS DESCARTES' ANGULAR DEFFICIENCY FORMULA. THE QUANTITY IN THE SUM IS THE SO CALLED SPHERICAL EXCESS AT A VERTEX. FOR INSTANCE, THE TETRAHEDRON HAS 3 ANGLES OF $\frac{\pi}{3}$ AT 4 VERTICES, AND SO WE ARE NOT SURPRISED THAT $4\cdot\left(2\pi - 3\cdot\frac{\pi}{3}\right) = 4\pi$

REARRANGING THE SUM AT RIGHT GIVES

$\sum_F (\text{S.ANGLE}) - \sum_F \pi n_f + \sum_F 2\pi = 2\pi V - 2\pi E + 2\pi F$

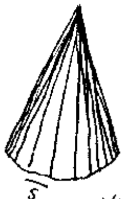
SO $\boxed{V + F = E + 2}$ WHICH IS CALLED EULER'S LAW.

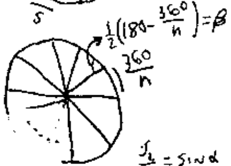


IF THE POLYHEDRON HAS h TUNNELS, IT IS ESSENTIALLY JUST LOSING $2h$ FACES SO $V + F = E + 2 - 2h$. THERE ARE MANY VARIATIONS ON EULER'S LAW THAT APPLY TO THE MANY DEFINITIONS OF A POLYHEDRON.

PYRAMIDS

SAY YOU WANT A PYRAMID WITH n SIDES AND YOU KNOW EITHER CORNER ANGLE α OR SIDE TILT γ SO

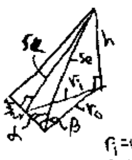




$\frac{s_c}{s_e}=\sin\alpha$ $\quad \frac{2r_i}{s}=\tan\beta$

$\frac{2s_c}{s}=\tan\alpha$ $\quad \frac{s}{2r_o}=\cos\beta$

$\frac{s}{2s_e}=\cos\alpha$ $\quad \frac{h}{r_i}=\tan\gamma$



r_i = INNER RADIUS
r_o = OUTER RADIUS

$$\sin\theta=\frac{\sin\beta}{\sin\alpha} \qquad \cos\theta=\sin\gamma\cos\beta$$

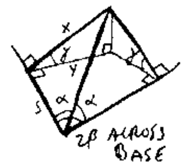
$$\boxed{\sin\theta=\frac{\cos\frac{180}{n}}{\sin\alpha}} \qquad \boxed{\cos\theta=\sin\gamma\sin\frac{180}{n}}$$

SO

$$\boxed{\frac{h}{s}=\frac{1}{2}\tan\gamma\tan\beta}$$
$$\boxed{\frac{r_i}{s}=\frac{1}{2}\tan\beta}$$
$$\boxed{\frac{r_o}{s}=\frac{1}{2\cos\beta}}$$
$$\boxed{\frac{s_c}{s}=\frac{1}{2}\tan\alpha}$$
$$\boxed{\frac{s_e}{s}=\frac{1}{2\cos\alpha}}$$

FROM THE PREVIOUS PAGE,

$$\boxed{\cos\theta=\sin\gamma\cos\beta}$$
$$\boxed{\sin\theta=\frac{\sin\beta}{\sin\alpha}}$$

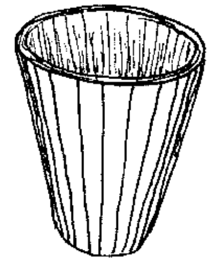


$\tan\alpha=\frac{x}{s}$

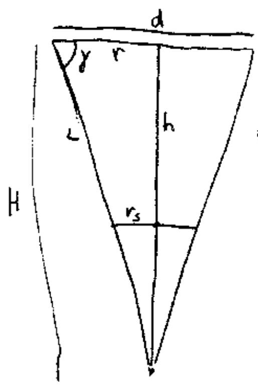
$\cos\gamma=\frac{y}{x}$

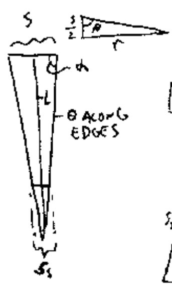
$\tan\beta=\frac{y}{s}$

SO $\boxed{\tan\alpha\cos\gamma=\tan\beta}$



TRUNCATED PYRAMIDS





SUPPOSE YOU KNOW γ, h, d, n

$$\boxed{\cos\theta=\sin\gamma\sin\frac{180}{n}}$$

$$s=2r\tan\frac{180}{n}$$

$$\boxed{s=d\tan\frac{180}{n}}$$

$$s_s=\frac{s(H-h)}{H}=s(1-\frac{h}{H})$$

$$\boxed{s_s=\tan\frac{180}{n}\left(d-\frac{2h}{\tan\gamma}\right)}$$

$$\boxed{L=\frac{h}{\sin\gamma}}$$

$$\boxed{\tan\alpha=\frac{1}{\cos\gamma\tan\frac{180}{n}}}$$

SUPPOSE INSTEAD YOU KNOW $\frac{s}{s_s}, h, d, n$

$$\boxed{s=d\tan\frac{180}{n}}$$

$$\boxed{s_s=\left(\frac{s_s}{s}\right)d\tan\frac{180}{n}}$$

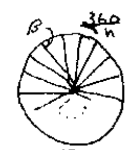
$$L^2=(r-r_s)^2+h^2=r^2\left(1-\frac{s_s}{s}\right)^2+h^2$$

$$\boxed{L=\sqrt{\frac{d^2}{4}\left(1-\left(\frac{s_s}{s}\right)\right)^2+h^2}}$$

$$\tan\gamma=\frac{h}{r-r_s}$$

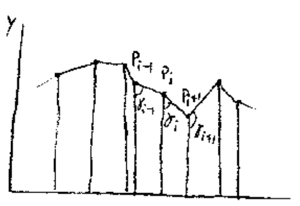
$$\boxed{\tan\gamma=\frac{2h}{d\left(1-\left(\frac{s_s}{s}\right)\right)}}$$

$$\boxed{\tan\alpha=\frac{\sqrt{1+\left(\frac{2h}{d\left(1-\frac{s_s}{s}\right)}\right)^2}}{\tan\frac{180}{n}}} \qquad \boxed{\sin\theta=\frac{\cos\frac{180}{n}}{\sin\alpha}}$$



$\beta=90-\frac{180}{n}$

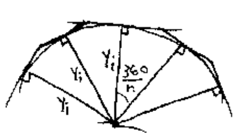
SOLIDS OF ROTATION

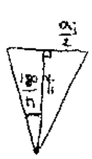


SAY YOU HAVE A COLLECTION OF POINTS $p_i=x_i,y_i$ WHERE p_i CONNECTS TO p_{i+1}. SPIN THIS ARRANGEMENT AROUND THE x AXIS AND DIVIDE INTO n SECTIONS EVENLY, HAVING THE POINT p_i DIVIDE AN EDGE IN HALF.

CALL $\beta=90-\frac{180}{n}$, AND WE SEE $\tan\gamma_i=\frac{x_{i+1}-x_i}{y_{i+1}-y_i}$

SO $\cos\theta_i=\cos\beta\sin\gamma_i$





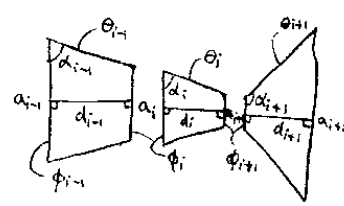
$$\boxed{\cos\theta_i=\sin\frac{180}{n}\frac{1}{\sqrt{1+\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)^2}}}$$

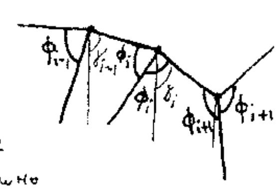
$$\boxed{\phi_i=\frac{1}{2}\left(180-\gamma_{i-1}+\gamma_i\right)}$$

$$\boxed{\tan\alpha_i=\frac{\sqrt{1+\left(\frac{x_{i+1}-x_i}{y_{i+1}-y_i}\right)^2}}{\tan\frac{180}{n}}}$$

$$\boxed{a_i=2y_i\tan\frac{180}{n}}$$

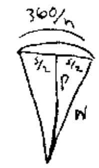
$$\boxed{d_i=\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}}$$





IT'S ABOUT THIS TIME YOU FETCH AN EAGER YOUNG PROGRAMMER, WHO WILL NO DOUBT BE ABLE TO WORK WONDERS FAR BEYOND THIS

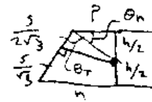
PRISMS AND ANTIPRISMS



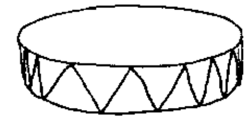
$$\frac{p}{N} = \cos\frac{180}{n}$$

$$\frac{2p}{s} = \frac{1}{\tan\frac{180}{n}}$$

$$\frac{2N}{s} = \frac{1}{\sin\frac{180}{n}}$$



$$h^2 + (N-p)^2 = \frac{3}{4}s^2$$



BOTH FIT WITHIN A SPHERE

ANTIPRISM WITH 2n TRIANGLES

$$\cos\theta_n = \frac{p}{\sqrt{\frac{h^2}{4}+p^2}} = \frac{1}{\sqrt{\left(\frac{h}{2p}\right)^2+1}} = \frac{1}{\sqrt{\frac{\frac{3}{4}\left(\frac{s}{p}\right)^2-\left(\frac{N}{p}-1\right)^2}{4}+1}}$$

$$= \frac{1}{\sqrt{\frac{3\tan^2\frac{180}{n}-\left(\frac{1}{\cos\frac{180}{n}}-1\right)^2}{4}+1}} = \frac{\sqrt{2}}{\sqrt{\tan^2\frac{180}{n}+\frac{1}{\cos\frac{180}{n}}+1}} = \frac{\sqrt{2}\cos\frac{180}{n}}{\sqrt{1+\cos\frac{180}{n}}} = \frac{\cos\frac{180}{n}}{\cos\frac{90}{n}}$$

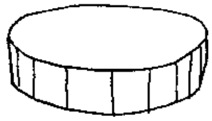
so $\boxed{\cos\theta_n = 2\cos\frac{90}{n} - \frac{1}{\cos 90/n}}$

$$\cos\theta_T = \frac{s/2\sqrt{3}}{\sqrt{\left(\frac{h}{2}\right)^2+p^2}} = \frac{1}{2\sqrt{3}\sqrt{\frac{1}{4}\left(\frac{h}{s}\right)^2+\left(\frac{p}{s}\right)^2}} = \frac{1}{2\sqrt{3}\sqrt{\frac{1}{4}\left(\frac{3}{4}-\left(\frac{N}{s}-\frac{p}{s}\right)^2\right)+\left(\frac{p}{s}\right)^2}}$$

$$= \frac{1}{\sqrt{3}\sqrt{\frac{3}{4}-\frac{1}{4}\left(\frac{1}{\sin\frac{180}{n}}-\frac{1}{\tan\frac{180}{n}}\right)^2+\frac{1}{\tan^2\frac{180}{n}}}} = \frac{1}{\sqrt{3}\sqrt{\frac{3}{4}+\frac{1}{\sin^2\frac{180}{n}}\left(-\frac{1}{4}-\frac{\cos^2\frac{180}{n}}{4}+\frac{\cos\frac{180}{n}}{2}+\cos^2\frac{180}{n}\right)}}$$

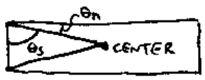
$$= \frac{2\sin\frac{180}{n}}{\sqrt{3}\sqrt{3-1+2\cos\frac{180}{n}}} = \sqrt{\frac{2}{3}}\frac{\sin\frac{180}{n}}{\sqrt{1+\cos\frac{180}{n}}}$$

so $\boxed{\cos\theta_T = \frac{2}{\sqrt{3}}\sin\frac{90}{n}}$



PRISM WITH n SQUARES

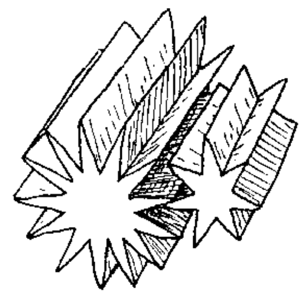
$$\boxed{\theta_s = 90 - \frac{180}{n}}$$
$$\boxed{\theta_n = \frac{180}{n}}$$



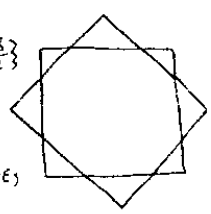
$\{\frac{5}{2}\}$

A POLYGON $\{\frac{p}{q}\}$ HAS p VERTICES AND SURROUNDS THE CENTER q TIMES.

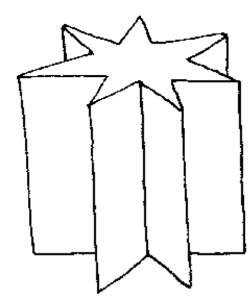
IF YOU ALLOW BOUNDING FACES THAT ARE STAR POLYGONS AND CONNECTING POLYGONS THAT RUN THROUGH EACH OTHER, THEN YOU ADMIT MORE INTERESTING PRISMS.



THIS MIGHT BE CALLED A $\{\frac{8}{2}\}$ WHICH REDUCES TO $\{4\}$ WHICH IS A SQUARE BECAUSE IT IS 2 COMPLETE SHAPES OVERLAID.



STRAIGHT PRISMS WITH SQUARES CAN ONLY GO STRAIGHT UP AND DOWN. THE ANTIPRISMS HAVE SLIGHTLY MORE FREEDOM. TRIANGLES CAN GO OUT, AS WITH NORMAL ANTIPRISMS, TO A LOWER POINT, OR, THE TRIANGLE CAN SOMETIMES FOLD THROUGH THE CENTER TO A POINT BELOW ON THE OPPOSITE SIDE. WITH THIS OPTION, BEWARE THAT THE RESULTING SHAPE IS PINCHED INTO 3 VOLUMES WHICH CONNECT ONLY ALONG SOME EDGES.



WITH POLYGON $\{\frac{p}{q}\}$ THE TABLE BELOW TELLS WHERE ANTIPRISM TRIANGLES CAN GO.

COINCIDENT

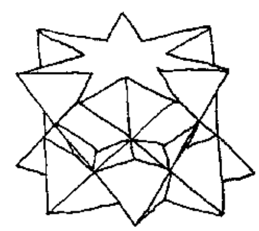
- q EVEN p ODD : OUT
- q ODD p EVEN : IMPOSSIBLE
- q ODD p ODD : ACROSS
- q EVEN p EVEN : EITHER

OFFSET

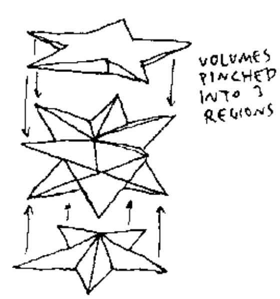
- q EVEN p ODD : ACROSS
- q ODD p EVEN : ACROSS
- q ODD p ODD : OUT
- q EVEN p EVEN : IMPOSSIBLE

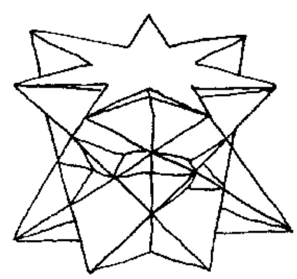
WHEN ACROSS, p AND q MUST SATISFY $\frac{p}{q} < 3$

OUT

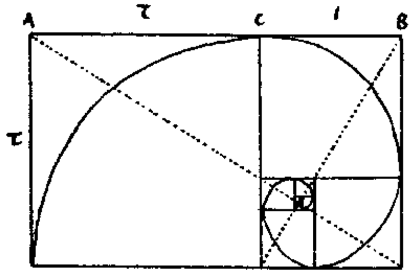


ACROSS





THE GOLDEN SECTION



A LOGARITHMIC SPIRAL TOUCHES THE POINT OF DIVISION ON EACH SUCCESSIVE RECTANGLE

A SEGMENT AB DIVIDED IN TWO AT C SUCH THAT $\frac{AB}{AC} = \frac{AC}{CB} = \tau$ IS SAID TO BE DIVIDED IN THE GOLDEN RATIO τ. SAY $CB = 1$ SO $\boxed{\tau^2 = \tau + 1}$ WHICH HAS ROOTS $\tau = \frac{1 \pm \sqrt{5}}{2}$. TAKE τ AS THE POSITIVE ROOT.

DIVIDE BY τ AND GET $\tau = 1 + \frac{1}{\tau}$

$$\text{SO} \quad \tau = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

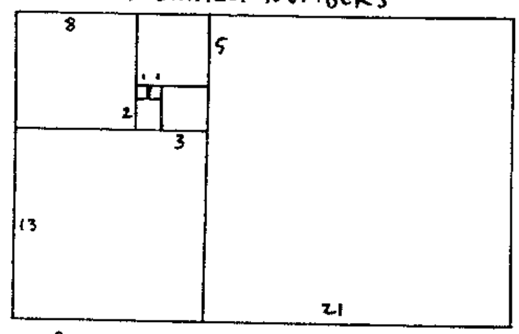
THIS PARTIALLY CONVERGES AS

$$1,\ 1 + \frac{1}{1} = 2,\ 1 + \frac{1}{2} = \frac{3}{2},\ 1 + \frac{1}{3/2} = \frac{5}{3}, \ldots$$

IN FACT, $1 + \frac{1}{a/b} = \frac{a+b}{a}$

SO THE n^{TH} CONVERGENCE IS $\frac{f_n}{f_{n-1}}$ WHERE f_n IS THE n^{TH} FIBONACCI NUMBER, WHICH IS DEFINED AS $f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$.

FIBONACCI NUMBERS

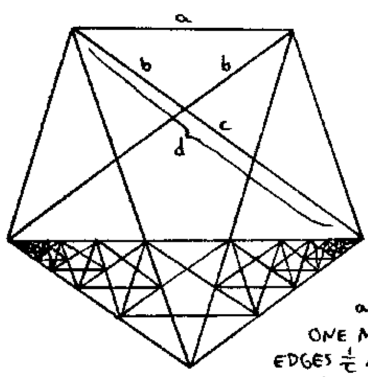


THUS $\lim_{n \to \infty} \frac{f_n}{f_{n-1}} = \tau$. THIS CAN BE REPRESENTED GRAPHICALLY AS FOLLOWS.

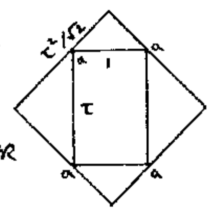
START WITH 2 UNIT SQUARES AND SUCCESSIVELY ADD SQUARES TO IT AS IN DIAGRAM. DO THIS AN INFINITE NUMBER OF TIMES, THEN TAKE ONE STEP BEYOND THE LARGEST RECTANGLE, TURN AROUND, AND GAZE SERENELY UPON THE GOLDEN RECTANGLE.

τ AND THE PENTAGON

LOOKING AT THE PENTAGON, ONE SEES FROM SIMILAR TRIANGLES THAT $\frac{d}{b+c} = \frac{a}{b} = \frac{b+c}{b}$ SO d IS DIVIDED IN THE GOLDEN SECTION, AND THOSE RATIOS EQUAL $\frac{b}{c} = \frac{d}{a} = \tau$, FROM WHICH $\cos 36 = \frac{a/2}{b} = \frac{\tau}{2}$, $\cos 72 = \frac{c/2}{b} = \frac{1}{2\tau}$, $\cos 36 \cos 72 = \frac{1}{4}$



τ AND POLYHEDRA



AROUND A GOLDEN RECTANGLE, DRAW A SQUARE. THREE SUCH SQUARES ARRANGED PERPENDICULAR TO EACH OTHER FORM AN OCTAHEDRON, AND ONE MAY QUICKLY SHOW THAT THE POINTS a ARE THE VERTICES OF AN ICOSAHEDRON. ONE MAY ALSO SHOW THAT RECTANGLES OF EDGES $\frac{1}{\tau}$ AND τ TOGETHER WITH A CUBE OF EDGE 1 MAKE THE VERTICES OF A DODECAHEDRON. THESE POINTS TOGETHER WITH THE POINTS a MAKE THE RHOMBIC TRIACONTAHEDRON, PICTURED LOWER RIGHT.

LET US NOW EXAMINE POWERS OF τ. SINCE $\tau^2 = \tau + 1$ AND $\tau(a\tau + b) = (a+b)\tau + a$,

$\boxed{\tau^n = f_n \tau + f_{n-1}}$ OR $\tau^{n+1} = f_{n+1}\tau + f_n$

ALSO SINCE $\frac{1}{\tau} = \tau - 1$, $\frac{1}{\tau}(a\tau - b) = -b\tau + (a+b)$, $a, b > 0$, AND $\frac{1}{\tau}(-a\tau + b) = b\tau - (a+b)$

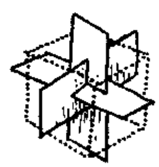
WE FIND $\boxed{\frac{1}{\tau^n} = (-1)^{n+1}(f_n \tau - f_{n+1})}$

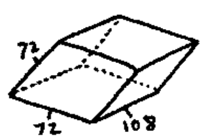
OR $(1-\tau)^n = f_{n+1} - f_n \tau$

NOW $\tau^n - (1-\tau)^n = \frac{\tau^{n+1}}{\tau} - (1-\tau)^n$

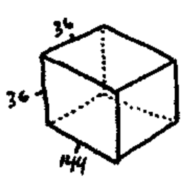
$$= \left(\tau + \frac{1}{\tau}\right) f_n = (2\tau - 1) f_n = \sqrt{5} f_n$$

SO $\boxed{f_n = \frac{\tau^n - (1-\tau)^n}{\sqrt{5}}}$





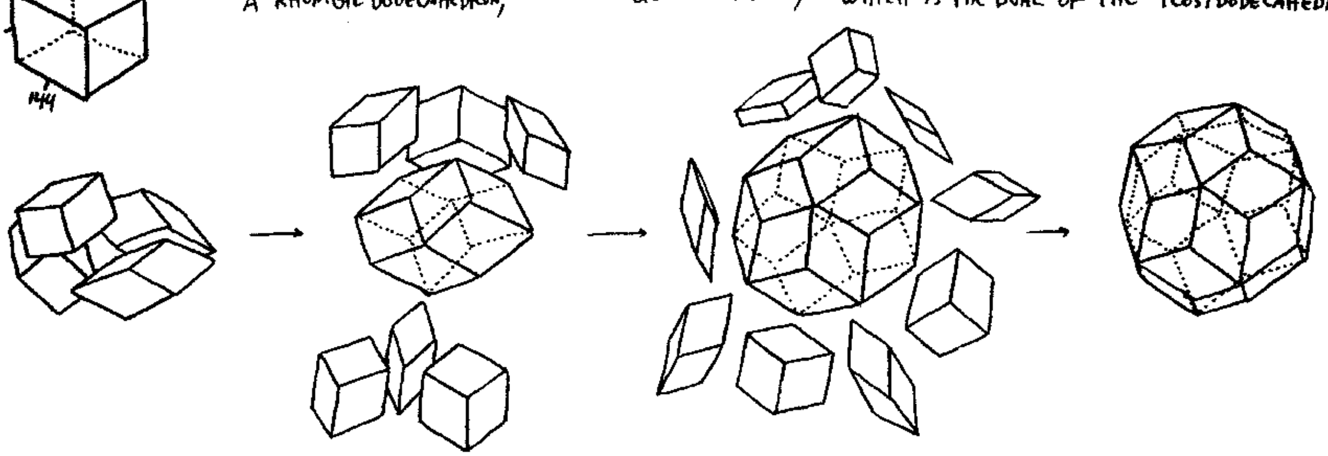
A RHOMBUS WHOSE DIAGONALS HAVE RATIO τ CAN BE PULLED OUT TO MAKE TWO DIFFERENT PARALLELEPIPEDS,



2 OF EACH KIND FIT TOGETHER TO MAKE A RHOMBIC DODECAHEDRON,

THREE MORE OF EACH ADDED TO THAT MAKE A RHOMBIC ICOSAHEDRON,

AND FIVE MORE OF EACH ADDED TO THAT MAKE THE RHOMBIC TRIACONTAHEDRON, WHICH IS THE DUAL OF THE ICOSIDODECAHEDRON.



PLATONIC SOLIDS

IN THE FOLLOWING,

i IS A RADIUS TO A FACE AND ALSO PERPENDICULAR TO IT

m IS A RADIUS TO AN EDGE AND PERPENDICULAR TO IT

o IS A RADIUS TO A VERTEX

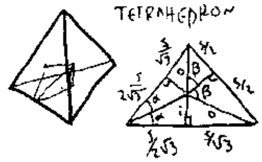
A SUBSCRIPT $_s$ REFERS TO THE EMBEDDED SOLID

θ IS AN ANGLE FROM A FACE TO THE CENTER

δ IS AN ANGLE FACE TO FACE (A DIHEDRAL ANGLE)

SUBSCRIPTS ON ANGLES REFER TO THE TYPE OF SHAPE WITH WHICH THE ANGLE IS ASSOCIATED, SO $_s$ WOULD MEAN SQUARE, NOT THE ANGLE ON AN EMBEDDED SOLID

TETRAHEDRON



$\sin\alpha = \frac{s/2}{s\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$\boxed{\alpha = 35.26438968}$

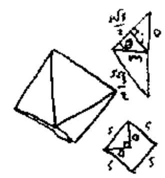
$\delta = 70.52877937$

$\frac{2i}{s} = \frac{1}{\sqrt{6}} = .4082482905$

$\frac{2m}{s} = \frac{1}{\sqrt{2}} = .7071067812$

$\frac{2o}{s} = \sqrt{\frac{3}{2}} = 1.224744871$

OCTAHEDRON



$\sin\beta = \frac{s/\sqrt{2}}{s\sqrt{3}/2} = \sqrt{\frac{2}{3}}$

$\boxed{\beta = 54.73561032}$

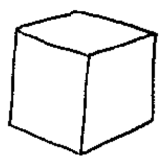
$\delta = 109.4712206$

$\frac{2i}{s} = \sqrt{\frac{2}{3}} = .8164965809$

$\frac{2m}{s} = 1$

$\frac{2o}{s} = \sqrt{2} = 1.414213562$

THE MOST COMPLACENT OF ALL SOLIDS, THE CUBE



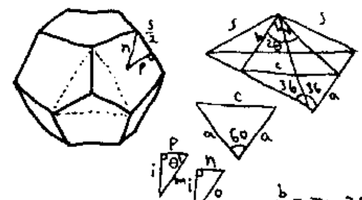
$\boxed{\theta_s = 45}$ $\delta = 90$

$\frac{2i}{s} = 1$

$\frac{2m}{s} = \sqrt{2} = 1.414213562$

$\frac{2o}{s} = \sqrt{3} = 1.732050808$

DODECAHEDRON



$\frac{b}{s} = \tan 36$

$\frac{s}{a} = \cos 36$

$c = a$

$\sin\theta = \frac{c/2}{b} = \frac{1}{2\sin 36} = \frac{\tau}{\sqrt{\tau+2}}$

$\boxed{\theta = 58.28252559}$

$\delta = 116.5650512$

$\frac{2i}{s} = \frac{i}{p}\frac{2p}{s} = \frac{\tan\theta}{\tan 36} = \frac{\tau^2}{\sqrt{\tau+2}} = 2.227032729$

$\frac{2m}{s} = \frac{\tan 54}{\cos\theta} = 4\cos^2 36 = \tau^2 = 2.618033989$

$\frac{2o}{s} = \sqrt{\left(\frac{2i}{s}\right)^2 + \left(\frac{2m}{s}\right)^2} = \sqrt{\frac{3(4\tau+3)}{\tau+2}} = 2.802517077$

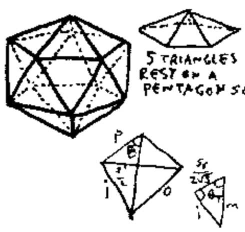
ALSO:

$\sin\theta = \frac{1}{2\sin 36} = \frac{\tau}{\sqrt{\tau+2}}$ $\sin 2\theta = \frac{2}{\tau(3-\tau)}$

$\cos\theta = \frac{1}{2\sin 72} = \frac{1}{\sqrt{\tau+2}}$ $\cos 2\theta = \frac{1}{\tau(\tau-1)}$

$\tan\theta = 2\cos 36 = \tau$ $\tan 2\theta = -2$

ICOSAHEDRON



$\sin\varepsilon = \frac{\sin 54}{\sin 60} = \frac{2\cos 36}{\sqrt{3}} = \frac{\tau}{\sqrt{3}}$

$\boxed{\varepsilon = 69.09484255}$

$\delta = 138.1896851$

$\frac{2i}{s} = \frac{\tan\theta}{\sqrt{3}} = \frac{4\cos^2 36}{\sqrt{3}} = \frac{\tau^2}{\sqrt{3}} = 1.511522628$

$\frac{2m}{s} = \frac{1}{\sqrt{4\sin^2 36 - 1}} = 2\cos 36 = \tau = 1.618033989$

$\frac{2o}{s} = 2\sin 72 = \sqrt{\tau+2} = 1.902113033$

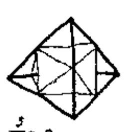
ALSO:

$\sin\varepsilon = \frac{\tau}{\sqrt{3}}$ $\sin 2\varepsilon = \frac{2}{3}$

$\cos\varepsilon = \frac{1}{\tau\sqrt{3}}$ $\cos 2\varepsilon = -\frac{\sqrt{5}}{3}$

$\tan\varepsilon = \tau^2$ $\tan 2\varepsilon = -\frac{2}{\sqrt{5}}$

OCTAHEDRON IN TETRAHEDRON



$\frac{s}{s_s} = 2$

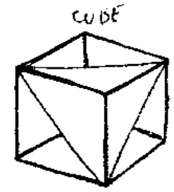
CUBE IN OCTAHEDRON

$\frac{s}{s_s} = \frac{3}{\sqrt{2}} = 2.121320344$

TETRAHEDRON IN TETRAHEDRON

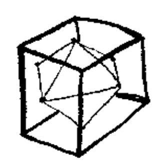
$\frac{s}{s_s} = 3$

TETRAHEDRON IN CUBE



$\frac{s}{s_s} = \frac{1}{\sqrt{2}} = .707106781$

OCTAHEDRON IN CUBE



$\frac{s}{s_s} = \sqrt{2} = 1.414213562$

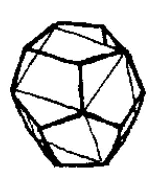
TETRAHEDRON IN DODECAHEDRON

$\frac{s}{s_s} = \frac{1}{\tau\sqrt{2}} = .4370160244$

ICOSAHEDRON IN OCTAHEDRON

$\frac{s}{s_s} = \frac{\tau^2}{\sqrt{2}} = 1.851229587$

CUBE IN DODECAHEDRON



$\frac{s}{s_s} = \frac{1}{\tau} = .6180339887$

DODECAHEDRON IN ICOSAHEDRON

$\frac{s}{s_s} = \frac{3}{\tau} = 1.854101966$

ICOSAHEDRON IN DODECAHEDRON

$\frac{s}{s_s} = \frac{3}{\tau} - 1 = .854101966$

CUBE IN OCTAHEDRON

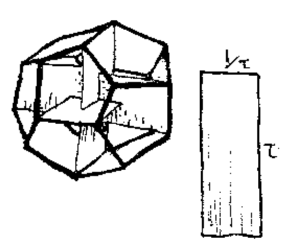
$\frac{s}{s_s} = \sqrt{2} = 1.4142136$

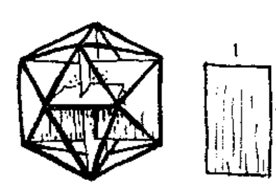
OCTAHEDRON IN ICOSAHEDRON

$\frac{s}{s_s} = \frac{\sqrt{2}}{\tau} = .8740320489$

OCTAHEDRON IN DODECAHEDRON

$\frac{s}{s_s} = \frac{\sqrt{2}}{\tau^2} = .5401815135$



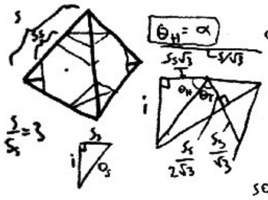


IN WHAT FOLLOWS,
$\theta = \theta_{\text{DODECAHEDRON}}$ $\alpha = \theta_{\text{TETRAHEDRON}}$
$\varepsilon = \theta_{\text{ICOSAHEDRON}}$ $\beta = \theta_{\text{OCTAHEDRON}}$

ARCHIMEDEAN SOLIDS

$\alpha = 35.26438968$
$\beta = 54.73561032$

GRIND THE CORNERS OF THE TETRAHEDRON A BIT AND GET: TRUNCATED TETRAHEDRON



$\frac{s}{s_s} = 3$

$\boxed{\theta_H = \alpha}$ $\cos\theta_T = \frac{s_s/2\sqrt{3}}{\frac{s_s\sqrt{3}}{2\cos\theta_{TET}}} = \frac{1}{3}\sqrt{\frac{2}{3}}$

$\boxed{\theta_T = 74.20683095}$

$\delta_{HH} = 70.52877937$
$\delta_{HT} = 109.4712206$
SO $\theta_H + \theta_T = 2\beta$

$\boxed{\begin{matrix}\theta_H = \alpha \\ \theta_T = 180 - 3\alpha\end{matrix}}$

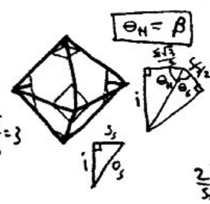
$\frac{2i_H}{s_s} = \sqrt{\frac{3}{2}} = 1.224744871$
$\frac{2m_s}{s_s} = \frac{3}{\sqrt{2}} = 2.121320344$
$\frac{2o}{s} = \sqrt{\frac{11}{2}} = 2.345207880$ $\frac{2i_T}{s_s} = \frac{5}{\sqrt{6}} = 2.041241452$

GRIND SOME MORE AND GET THE OCTAHEDRON

FROM PREVIOUS PAGE WE SEE $\boxed{\alpha + \beta = 90}$

$\frac{s}{s_s} = 2$

GRIND THE OCTAHEDRON'S CORNERS AND SO FORM THE TRUNCATED OCTAHEDRON



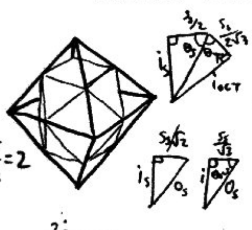
$\frac{s}{s_s} = 3$

$\boxed{\theta_H = \beta}$ $\cos\theta_s = \frac{s_s/2}{\frac{s_s\sqrt{3}}{2\cos\theta_H}} = \frac{1}{3}$ OR $\boxed{\begin{matrix}\theta_s + 2\theta_H = 180 \\ \theta_s = 2\alpha\end{matrix}}$

$\boxed{\theta_s = 70.52877937}$
$\delta_{HH} = 109.4712206$
$\delta_{HS} = 125.2643897$

$\frac{2i_H}{s_s} = \sqrt{6} = 2.449489743$
$\frac{2m}{s_s} = 3$
$\frac{2i_s}{s_s} = 2\sqrt{2} = 2.828427125$ $\frac{2o}{s_s} = \sqrt{10} = 3.16227766$

GRIND AWAY AND PRODUCE THE CUBOCTAHEDRON

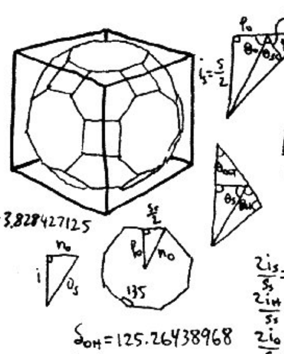


$\frac{s}{s_s} = 2$

$\tan\theta_T = \frac{2i}{s} \cdot \frac{s\sqrt{3}}{s_s} = \sqrt{\frac{2}{3}} \cdot 2\sqrt{3} = 2\sqrt{2}$
$\boxed{\theta_T = 70.52877937}$
$\cos\theta_s = \frac{s_s/2}{\frac{s_s}{2\sqrt{3}\cos\theta_T}} = \frac{1}{\sqrt{3}}$
$\boxed{\theta_s = 54.73561032}$
$\boxed{\begin{matrix}\theta_T = 2\alpha \\ \theta_s = \beta\end{matrix}}$
$\delta_{ST} = 125.2643893$

$\frac{2i_T}{s_s} = 2\sqrt{\frac{2}{3}} = 1.632993162$
$\frac{2i_s}{s_s} = \sqrt{2} = 1.414213562$
$\frac{2m_s}{s_s} = \sqrt{3} = 1.732050808$
$\frac{2o_s}{s_s} = 2$

GRIND DOWN THE CORNERS AND THE TRIANGLES A BIT AND IT SPAWNS THE TRUNCATED CUBOCTAHEDRON



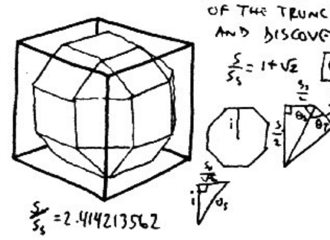
$\frac{s}{s_s} = 3.828427125$

$\frac{s}{2} = \frac{s_s}{2}\tan 67.5 + s_s\frac{1}{\sqrt{2}}$
$\frac{s}{s_s} = \tan 67.5 + \sqrt{2} = 1 + 2\sqrt{2}$
$\tan\theta_o = \frac{s}{2p_o} = 1 + \sqrt{2}\tan 22.5 = \frac{1+2\sqrt{2}}{1+\sqrt{2}}$
$\boxed{\theta_o = 57.76438968} = 22.5 + \alpha$
$\boxed{\theta_s = 135 - \theta_o} = 77.23561032 = 22.5 + \beta$
$\theta_s + \theta_H - \beta = 90$
$\boxed{\theta_H = 67.5}$

$\delta_{OH} = 125.26438968$
$\delta_{OS} = 135$
$\delta_{HS} = 144.73561032$

$\frac{2i_s}{s_s} = 3 + \sqrt{2} = 4.414213562$
$\frac{2i_H}{s_s} = \sqrt{3}(1+\sqrt{2}) = 4.18154055$
$\frac{2i_o}{s_s} = 1 + 2\sqrt{2} = 3.828427125$
$\frac{2m_s}{s_s} = \sqrt{6}\sqrt{2+\sqrt{2}} = 4.526066877$
$\frac{2o_s}{s_s} = \sqrt{13+6\sqrt{2}} = 4.635221826$

GRIND THE SQUARES AND HEXAGONS OF THE TRUNCATED CUBOCTAHEDRON AND DISCOVER THE RHOMBICUBOCTAHEDRON

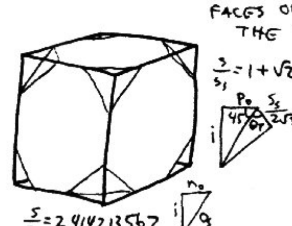


$\frac{s}{s_s} = 2.414213562$

$\frac{s}{s_s} = 1 + \sqrt{2}$ $\boxed{\theta_s = 67.5}$
$\cos\theta_T = \frac{s_s/2\sqrt{3}}{\frac{s_s}{2\cos 67.5}} = \frac{\cos 67.5}{\sqrt{3}}$
$\boxed{\theta_T = 77.23561032} = 22.5 + \beta$
$\delta_{SS} = 135$
$\delta_{ST} = 90 + \beta = 144.73561032$

$\frac{2i_s}{s_s} = 1 + \sqrt{2} = 2.414213562$ $\frac{2m_s}{s_s} = \sqrt{2}\sqrt{2+\sqrt{2}} = 2.61312593$
$\frac{2i_T}{s_s} = \frac{3+\sqrt{2}}{\sqrt{3}} = 2.548547388$ $\frac{2o_s}{s_s} = \sqrt{5+2\sqrt{2}} = 2.797932652$

GRIND THE FACES PARALLEL TO THE FACES OF THE CUBE AND REVEAL THE TRUNCATED CUBE

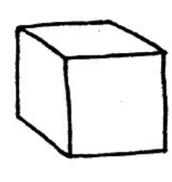


$\frac{s}{s_s} = 2.414213562$

$\frac{s}{s_s} = 1 + \sqrt{2}$, $\boxed{\theta_o = 45}$
$\cos\theta_T = \frac{s_s/2\sqrt{3}}{s/\sqrt{2}} = \frac{s_s}{s\sqrt{6}} = \frac{1}{(1+\sqrt{2})\sqrt{6}}$
$\boxed{\theta_T = 80.26438968} = 45 + \alpha$
$\delta_{oo} = 90$
$\delta_{oT} = 125.26438968$

$\frac{2i_T}{s_s} = \frac{\sqrt{3}}{2}(2+\sqrt{2}) = 2.956795679$
$\frac{2i_o}{s_s} = 1 + \sqrt{2} = 2.414213562$
$\frac{2m_s}{s_s} = 2 + \sqrt{2} = 3.414213562$
$\frac{2o_s}{s_s} = \sqrt{7+4\sqrt{2}} = 3.557647291$

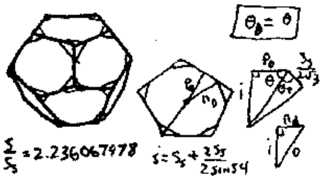
GRIND AWAY THE OCTAGONS AND HAPPEN UPON THE MOST INNOCUOUS YET UBIQUITOUS OF ALL SOLIDS, THE CUBE



$\boxed{\theta = 58.28252559}$

$\boxed{\varepsilon = 69.09484255}$

GRIND THE CORNERS OF THE DODECAHEDRON AND DELIVER THE TRUNCATED DODECAHEDRON



$\boxed{\theta_D = \theta}$

$\cos\theta_T = \frac{s_s/2\sqrt{3}}{\rho_D/\cos\theta} = \frac{1}{\sqrt{3}(3\tau+1)}$

$\boxed{\theta_T = 84.34010627} = 270 - 2\theta - \varepsilon$

$\frac{s}{s_s} = 2.236067978$ $\quad s = s_s + \frac{2s_T}{2\sin 54}$ $\quad \frac{s}{s_s} = \sqrt{5}$

$\delta_{DD} = 116.5650512$

$\delta_{DT} = 142.6226319$

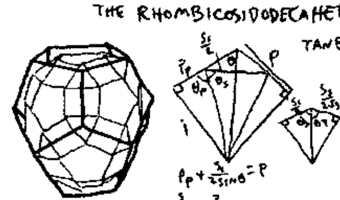
$\frac{2i_D}{s_s} = \sqrt{11\tau+7} = 4.97979657$

$\frac{2i_T}{s_s} = \sqrt{15\tau+\frac{29}{3}} = 5.852556233$

$\frac{2m}{s_s} = 3\tau+1 = 5.854101966$

$\frac{2o_s}{s_s} = \sqrt{15\tau+11} = 5.938898032$

GRIND THE HEXAGONS AND SQUARES SOME AND UNMASK THE RHOMBICOSIDODECAHEDRON



$\tan\theta_P = \frac{i}{\rho_P} = 3$

$\boxed{\theta_P = 71.56505118} = 2\theta - 45$

$\theta_S + \theta_P = 90 + \theta$ so $\theta_S = 135 - \theta$

$\boxed{\theta_S = 76.71747441}$

$\cos\theta_T = \frac{s_s/2\sqrt{3}}{s_s/2\cos\theta_S} = \frac{1}{\sqrt{30}}\sqrt{7-4\tau}$

$\boxed{\theta_T = 82.37736814} = \theta + \varepsilon - 45$

$\frac{s}{s_s} = 1.854101966$ $\quad \rho_P + \frac{s_s}{2\sin\theta} = \rho$ $\quad \frac{s}{s_s} = \frac{3}{\tau}$

$\frac{2i_P}{s_s} = \frac{3}{\sqrt{5}}\sqrt{4\tau+3} = 4.129145761$

$\frac{2i_S}{s_s} = \tau^3 = 4.236067977$

$\frac{2i_T}{s_s} = \frac{1}{\sqrt{3}}\sqrt{24\tau+17} = 4.314039705$

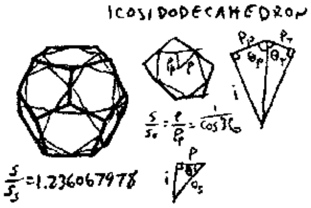
$\frac{2m_s}{s_s} = \sqrt{2}\sqrt{4\tau+3} = 4.352501799$

$\frac{2o_s}{s_s} = \sqrt{8\tau+7} = 4.465901019$

$\delta_{PS} = 148.2825256$

$\delta_{ST} = 159.0948426$

GRIND SOME MORE AND PROCURE THE ICOSIDODECAHEDRON



$\tan\theta_P = \frac{i}{\rho_P} = \frac{i}{\rho}\frac{\rho}{\rho_P} = 2$

$\boxed{\theta_P = 63.43494882} = 180 - 2\theta$

$\cos\theta_T = \frac{s_s/2\sqrt{3}}{\rho_P/\cos\theta_P} = \frac{\tan 36}{\sqrt{15}} = \frac{1}{\sqrt{15}}\sqrt{7-4\tau}$

$\boxed{\theta_T = 79.18768304} = 90 + \theta - \varepsilon$

$\frac{s}{s_s} = 1.236067978$ $\quad \frac{s}{s_s} = \frac{\rho}{\rho_P} = \frac{1}{\cos 36}$

$\delta_{PT} = 142.6226319 = 270 - \theta - \varepsilon$

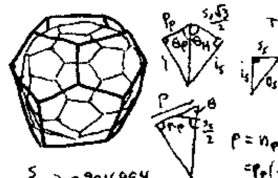
$\frac{2i_T}{s_s} = \frac{2}{\sqrt{3}}\tau^2 = 3.023045256$

$\frac{2i_P}{s_s} = \frac{2}{\sqrt{5}}\sqrt{4\tau+3} = 2.752763841$

$\frac{2m_s}{s_s} = \sqrt{4\tau+3} = 3.077683537$

$\frac{2o_s}{s_s} = 2\tau = 3.236067978$

GRIND THE TRIANGLES AND THE PENTAGONS AND SO SCULPT THE TRUNCATED ICOSAHEDRON



$\tan\theta_P = \frac{i}{\rho_P} = \tan\theta\frac{s}{s_s} = 5 - \tau$

$\boxed{\theta_P = 73.52778951} = 270 - 2\varepsilon - \theta$

BY CORNER FORMULA DERIVED EARLIER

$\sin\theta_H = \frac{\sin 54}{\sin 120} = \frac{2\cos 36}{\sqrt{3}} = \frac{\tau}{\sqrt{3}}$

$\boxed{\theta_H = 69.09484255} = \varepsilon$

$\frac{s}{s_s} = 2.09016994$ $\quad \rho = \rho_P + \frac{s_s}{2\sin\theta} = \rho_P\left(\frac{1+2\sin^2 36}{\cos 36}\right)$ $\quad \frac{s}{s_s} = 5\tau - 6$

$\delta_{HH} = 138.1896851$

$\delta_{HP} = 142.6226319$

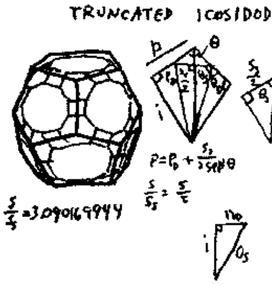
$\frac{2i_H}{s_s} = \sqrt{3}\tau^2 = 4.534567884$

$\frac{2i_P}{s_s} = \frac{1}{\sqrt{5}}\sqrt{41\tau+42} = 4.654876874$

$\frac{2m_s}{s_s} = 3\tau = 4.854101966$

$\frac{2o_s}{s_s} = \sqrt{9\tau+10} = 4.956037318$

GRIND THE CORNERS AND THE TRIANGLES A BIT AND FIND THE TRUNCATED ICOSIDODECAHEDRON



$\tan\theta_D = \frac{i}{\rho_D} = \sqrt{5}$

$\boxed{\theta_D = 65.90515745} = 135 - \varepsilon$

$\theta_S + \theta_D = 90 + \theta$

$\tan\theta_S = 4\tau + 1$

$\boxed{\theta_S = 82.37736814} = \theta + \varepsilon - 45$

$\cos\theta_H = \frac{s_s\sqrt{3}/2}{\sqrt{i^2+\rho_D^2}} = \frac{1}{\sqrt{10}}\sqrt{7-4\tau}$

$\boxed{\theta_H = 76.71747441} = 135 - \theta$

$\frac{s}{s_s} = 3.090169944$ $\quad \rho = \rho_D + \frac{s_s}{2\sin\theta}$ $\quad \frac{s}{s_s} = \frac{5}{\tau}$

$\frac{2i_D}{s_s} = \sqrt{5}\sqrt{4\tau+3} = 6.881909602$

$\frac{2i_H}{s_s} = \sqrt{3}\tau^3 = 7.337084961$

$\frac{2i_S}{s_s} = 4\tau+1 = 7.472135955$

$\frac{2m_s}{s_s} = \sqrt{6}\sqrt{4\tau+3} = 7.538754256$

$\frac{2o_s}{s_s} = \sqrt{24\tau+19} = 7.604789000$

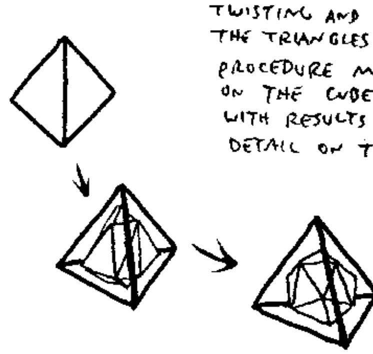
$\delta_{DH} = 142.6226319$

$\delta_{DS} = 148.2825256$

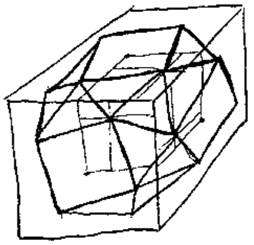
$\delta_{HS} = 159.0948426$

GRIND THE HEXAGONS UNTIL YOU EXHUME THE ICOSAHEDRON, BUT ENOUGH OF THAT FOR NOW.

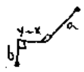
TOPOLOGICALLY SPEAKING, ONE MAY ENVISION THE ICOSAHEDRON AS A SNUB TETRAHEDRON, THAT IS, SHRINKING EACH SIDE AND TWISTING, FILLING IN THE GAPS WITH TRIANGLES, KEEP TWISTING AND SHRINKING UNTIL ALL THE TRIANGLES ARE THE SAME. THIS PROCEDURE MAY ALSO BE PERFORMED ON THE CUBE AND THE DODECAHEDRON, WITH RESULTS SHOWN IN FAR TOO MUCH DETAIL ON THE NEXT PAGE.



THE SNUB CUBE



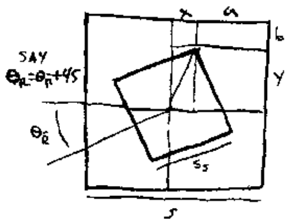
S_s SPANS BOTH:



$$s_s^2=(2x)^2+2b^2$$
$$=(\sqrt{2}s_s\cos\theta_R)^2+2\left(\frac{s}{2}-\frac{s_s\sin\theta_R}{\sqrt{2}}\right)^2$$
$$=2s_s^2\cos^2\theta_R+\frac{s^2}{2}-\sqrt{2}\,ss_s\sin\theta_R+s_s^2\sin^2\theta_R$$
$$1=2\cos^2\theta_R+\frac{1}{2}\left(\frac{s}{s_s}\right)^2-\sqrt{2}\left(\frac{s}{s_s}\right)\sin\theta_R+\sin^2\theta_R$$
$$0=\cos^2\theta_R+\frac{1}{2}\left(\frac{s}{s_s}\right)^2-\sqrt{2}\left(\frac{s}{s_s}\right)\sin\theta_R$$
$$0=\left(\frac{s}{s_s}\right)^2-2\sqrt{2}\sin\theta_R\left(\frac{s}{s_s}\right)+2\cos^2\theta_R$$
$$\hookrightarrow \frac{s}{s_s}=\sqrt{2}\left(\sin\theta_R\pm\sqrt{2\sin^2\theta_R-1}\right)$$

MUST TAKE POSITIVE ROOT FOR $\frac{s}{s_s}>1$

$$s_s^2=a^2+b^2+(y-x)^2$$
$$=\left(\frac{s}{2}-x\right)^2+\left(\frac{s}{2}-y\right)^2+(y-x)^2$$
$$=\frac{s^2}{2}+2x^2+2y^2-2xy-sx-sy$$
$$=\frac{s^2}{2}+2\left(\frac{s_s^2}{2}\cos^2\theta_R+\frac{s_s^2}{2}\sin^2\theta_R\right)-2\frac{s_s^2}{2}\cos\theta_R\sin\theta_R-\frac{ss_s}{\sqrt{2}}(\cos\theta_R+\sin\theta_R)$$
$$=\frac{s^2}{2}+s_s^2-s_s^2\cos\theta_R\sin\theta_R-\frac{ss_s}{\sqrt{2}}(\cos\theta_R+\sin\theta_R)$$
$$1=\frac{1}{2}\left(\frac{s}{s_s}\right)^2+1-\cos\theta_R\sin\theta_R-\frac{1}{\sqrt{2}}\left(\frac{s}{s_s}\right)(\cos\theta_R+\sin\theta_R)$$
$$0=\left(\frac{1}{\sqrt{2}}\left(\frac{s}{s_s}\right)\right)^2-\cos\theta_R\sin\theta_R-\frac{1}{\sqrt{2}}\left(\frac{s}{s_s}\right)(\cos\theta_R+\sin\theta_R)$$



$$x=\frac{s_s\cos\theta_R}{\sqrt{2}}$$
$$y=\frac{s_s\sin\theta_R}{\sqrt{2}}$$

PRETEND NOW THAT $x=\cos\theta_R$, $y=\sin\theta_R$ SO

$$0=\left(y+\sqrt{2y^2-1}\right)^2-xy-\left(y+\sqrt{2y^2-1}\right)(x+y)=2y^2-1-2xy+(y-x)\sqrt{2y^2-1}\quad\text{OR}\quad\sqrt{2y^2-1}\,(y-x)=1+2xy-2y^2$$

SO SQUARE AND GET $2y^4-4xy^3+2x^2y^2-y^2+2xy-x^2=4y^4-2y^2-4xy^3-2y^2+1+2xy-4xy^3+2xy+4x^2y^2$

$$3y^2=2y^4+2x^2y^2+x^2+1-4xy^3+2xy$$
$$3\sin^2\theta_R=2\sin^4\theta_R+2\cos^2\theta_R\sin^2\theta_R+\cos^2\theta_R+1-4\cos\theta_R\sin^3\theta_R+2\cos\theta_R\sin\theta_R$$
$$2-4\cos^2\theta_R=2\sin^2\theta_R-2\cos\theta_R\sin\theta_R+4\cos^3\theta_R\sin\theta_R$$
$$0=2\cos\theta_R-4\sin^3\theta_R+2\sin\theta_R$$
$$=\cos\theta_R-\sin\theta_R(2\sin^2\theta_R-1)$$
$$=\cos\theta_R+\sin\theta_R\cos 2\theta_R\quad\text{SO}\quad\cos 2\theta_R=\frac{-\cos\theta_R}{\sin\theta_R},\ \text{SO}\ \sin^2\theta_R=\frac{1}{6}\left(\sqrt[3]{17+3\sqrt{33}}+\sqrt[3]{17-3\sqrt{33}}\right)+\frac{1}{3}$$

$\boxed{\theta_R=61.46756039°}$ SO SQUARE IS OFFSET $\boxed{16.46756039}$

SO $\boxed{\frac{s}{s_s}=2.285227017}$

$\boxed{\frac{s_s}{s}=.437593284}$

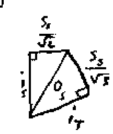
$$\tan\theta_s=\frac{s}{s_s}$$

SO $\boxed{\theta_s=66.76613621}$

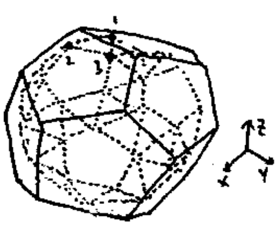
$$\cos\theta_T=\frac{s/2\sqrt{3}}{\sqrt{\left(\frac{s}{2}\right)^2+\left(\frac{s_s}{2}\right)^2}}=\frac{1}{\sqrt{3}\sqrt{\left(\frac{s}{s_s}\right)^2+1}}$$

SO $\boxed{\theta_T=76.61729385}$

$$\frac{2i_s}{s_s}=\frac{s}{s_s}=2.285227017$$
$$\frac{2o_s}{s_s}=\sqrt{\left(\frac{2i_s}{s_s}\right)^2+2}=2.687426747$$
$$\frac{2i_T}{s_s}=\sqrt{\left(\frac{s}{s_s}\right)^2+\frac{2}{3}}=2.42671160$$
$$\frac{2P}{s_s}=\sqrt{\left(\frac{s}{s_s}\right)^2+1}=2.49444634$$



THE SNUB DODECAHEDRON

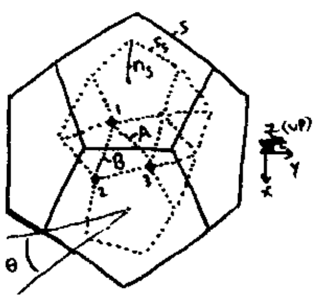


$$A=2\sqrt{(\vec{3}_x)^2+(\vec{3}_y)^2}$$

SO
$$\frac{A^2}{4}=\sin^2(\theta-36)n_s^2+\frac{\tau^2}{5}+n_s^2\frac{\cos^2(\theta-36)}{4\sin^2 36}-n_s\frac{\tau}{\sqrt{5}}\frac{\cos(\theta-36)}{\sin 36}$$

$$B=\sqrt{(2_x+3_x)^2+(2_y+3_y)^2+(3_z-2_z)^2}$$

SO
$$B^2=\frac{4}{5}\tau^2-n_s\cos\theta\left(\frac{4\tau^3}{\sqrt{5}\sqrt{\tau^2+1}}\right)+n_s^2\left(2+\frac{\tau^2}{2}-\frac{1}{2\sqrt{5}}\left(1+\frac{1}{\tau^2}\right)-\frac{4}{\sqrt{5}\tau}\cos^2\theta\right)$$



A=B EVENTUALLY RESULTS IN:

$$\frac{s}{s_s}=1.77897336$$
$$\theta=13.10640338$$

I LEAVE IT AS AN EXERCISE FOR THE READER TO FIND EXACT EXPRESSIONS FOR $\frac{s}{s_s}$ AND θ.

SAY $s=\frac{2}{\tau}$

$$\vec{1}=(-\vec{3}_x,-\vec{3}_y,\vec{3}_z)$$
$$\vec{2}=n_s\left(-\frac{\cos(36+\theta)}{2\sin 36}\right)+\frac{\tau}{\sqrt{5}},$$
$$n_s\left(-\sin(36+\theta)\right),$$
$$n_s\left(\frac{\cos(36+\theta)}{2\sin 72}\right)+\tau-\frac{1}{\sqrt{5}}$$
$$\vec{3}=n_s\left(-\frac{\cos(\theta-36)}{2\sin 36}\right)+\frac{\tau}{\sqrt{5}},$$
$$n_s\left(-\sin(\theta-36)\right),$$
$$n_s\left(\frac{\cos(\theta-36)}{2\sin 72}\right)+\tau-\frac{1}{\sqrt{5}}$$

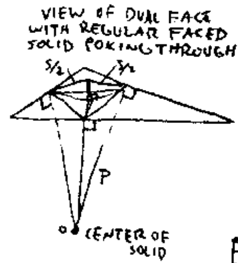
$$\tan\theta_P=\frac{i_P}{P_s}=2\cos 36\left(\frac{s}{s_s}\right)\quad\boxed{\theta_P=70.8422373}$$

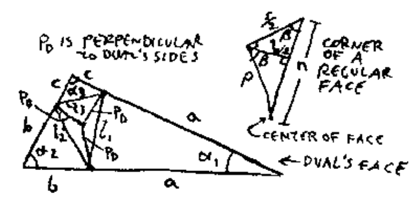
$$\cos\theta_T=\frac{s_s/2\sqrt{3}}{P_s/\cos\theta_P}=\frac{\tan 36}{\sqrt{3}\sqrt{1+4\cos^2 36\left(\frac{s}{s_s}\right)^2}}\quad\boxed{\theta_T=82.0876830}$$

$$\frac{2i_P}{s_s}=\frac{2i_P}{P_s}\frac{P_s}{s_s}=\frac{2\cos 36}{\tan 36}\left(\frac{s}{s_s}\right)=3.96183189$$
$$\frac{2P}{s_s}=\sqrt{\left(\frac{2i_P}{s_s}\right)^2+\frac{1}{\tan^2 36}}=4.19410767$$
$$\frac{2i_T}{s_s}=\sqrt{\left(\frac{2P}{s_s}\right)^2-\frac{1}{3}}=4.15417932$$
$$\frac{2O}{s_s}=\sqrt{\left(\frac{2i_P}{s_s}\right)^2+\frac{1}{\sin^2 36}}=4.48905703$$

DUALS

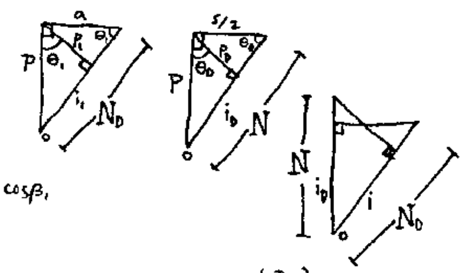
EACH FACE OF A SOLID HAS A CORRESPONDING VERTEX IN ITS DUAL, AND EACH VERTEX, A FACE. THEY BOTH HAVE THE SAME NUMBER OF EDGES, WHICH INTERSECT AT RIGHT ANGLES WHEN THEY HAVE THE SAME INTERSPHERE.





INTERSPHERE — A SPHERE TANGENT TO ALL THE EDGES IN A SOLID. IN PLATONIC AND ARCHIMEDEAN SOLIDS THE POINT OF TANGENCY IS THE MIDPOINT OF EACH EDGE

- l — DISTANCE ON A FACE FROM POINT OF TANGENCY OF INTERSPHERE TO SAME ON ADJACENT EDGE
- S — LENGTH OF SIDE ON PLATONIC OR ARCHIMEDEAN SOLID
- $S_{1,2,\ldots}$ — LENGTH OF SIDES OF FACE OF ARCHIMEDEAN DUALS
- $\theta_{1,2,3,\ldots}$ — ANGLE FROM FACE TO CENTER OF PLATONIC OR ARCHIMEDEAN SOLID
- θ_D — HALF THE DIHEDRAL ANGLE OF AN ARCHIMEDEAN DUAL
- i — SHORTEST DISTANCE FROM CENTER TO A FACE
- N — SHORTEST DISTANCE FROM CENTER TO A VERTEX
- P — SHORTEST DISTANCE FROM CENTER TO POINT OF TANGENCY OF INTERSPHERE
- ρ — SHORTEST DISTANCE ON A FACE FROM FACE'S CENTER TO POINT OF TANGENCY OF INTERSPHERE
- n — SHORTEST DISTANCE ON A FACE FROM FACE'S CENTER TO A CORNER
- β — HALF THE CORNER ANGLE OF A REGULAR FACE



$\frac{l_1}{2} = \rho_1 \cos\beta_1$

$a = \frac{P}{i_1}\rho_1 = \frac{\rho_1}{\sin\theta_1}$

$\sin\frac{\alpha_1}{2} = \frac{l_1/2}{a} = \sin\theta_1\cos\beta_1$

$\tan\theta_D = \frac{P}{S/2} = \frac{2P}{S}$

$\tan\theta_1 = \frac{P}{a}$

SAY $S_1 = a+b$, THEN

$\frac{S_1}{S} = \frac{a+b}{S/2+S/2} = \frac{\tan\theta_D}{2}\left(\frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2}\right)$

$\frac{2P}{S_1} = \frac{2P}{a+b} = \frac{2}{\frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2}}$

$\frac{2i_D}{S_1} = \frac{2P}{S_1}\frac{i_D}{P} = \left(\frac{2P}{S_1}\right)\sin\theta_D = \frac{2\sin\theta_D}{\frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2}}$

$\frac{2N_D}{S_1} = \frac{2Pa}{S_1\rho_1} = \frac{2}{\left(1+\frac{\sin\theta_1\,\rho_2}{\sin\theta_2\,\rho_1}\right)\cos\theta_1}$

$\sin\theta_D = \frac{(2P/S)}{(2N/S)}$

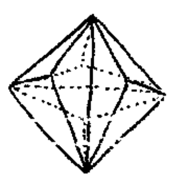
$\frac{N_D}{i_D} = \frac{N}{i}$

$\frac{2N_D}{S_1} = \left(\frac{2P}{S_1}\right)\frac{1}{\sin\theta_1}$

$\frac{2i_D}{S_1} = \left(\frac{2P}{S}\right)\left(\frac{2P}{S_1}\right)\frac{1}{(2N/S)}$

$\frac{S}{S_1} = \frac{(2P/S_1)}{(2P/S)}$

TRIAKIS TETRAHEDRON



$\tan\theta_D = \frac{3}{\sqrt{2}}$, $\theta_D = 64.7605982$

$\sin\frac{\alpha_1}{2} = \frac{5}{6}$, $\alpha_1 = 112.8853805$

$\sin\frac{\alpha_2}{2} = \frac{1}{2\sqrt{3}}$, $\alpha_2 = 33.5573098$

$\frac{2i_D}{S_1} = \frac{3}{\sqrt{22}} = .639602149$

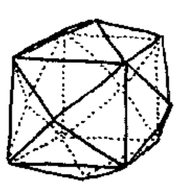
$\frac{2P}{S_1} = \frac{1}{\sqrt{2}} = .707106781$

$\frac{2N_3}{S_1} = \frac{3}{5}\sqrt{\frac{3}{2}} = .734846923$

$\frac{2N_6}{S_1} = \sqrt{\frac{3}{2}} = 1.22474487$

$\frac{S_1}{S} = 3$

TETRAKIS HEXAHEDRON



$\tan\theta_D = 3$, $\theta_D = 71.5650512 = 2\theta - 45$

$\sin\frac{\alpha_1}{2} = \frac{2}{3}$, $\alpha_1 = 83.6206298$

$\sin\frac{\alpha_2}{2} = \frac{1}{\sqrt{6}}$, $\alpha_2 = 48.1896851$

$\frac{2i_D}{S_1} = \frac{3}{\sqrt{5}} = 1.34164079$

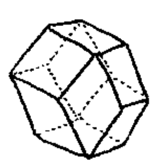
$\frac{2P}{S_1} = \sqrt{2} = 1.41421356$

$\frac{2N_4}{S_1} = \frac{3}{2}$

$\frac{2N_6}{S_1} = \sqrt{3} = 1.73205081$

$\frac{S_1}{S} = \frac{3}{\sqrt{2}} = 2.12132034$

RHOMBIC DODECAHEDRON



$\theta_D = 60$

$\sin\frac{\alpha_1}{2} = \frac{1}{\sqrt{3}}$, $\alpha_1 = 70.5287794$

$\sin\frac{\alpha_2}{2} = \frac{\sqrt{2}}{\sqrt{3}}$, $\alpha_2 = 109.4712206$

$\frac{2i_D}{S_1} = 2\sqrt{\frac{2}{3}} = 1.63299316$

$\frac{2P}{S_1} = \frac{4\sqrt{2}}{3} = 1.88561808$

$\frac{2N_3}{S_1} = 2$

$\frac{2N_4}{S_1} = \frac{4}{\sqrt{3}} = 2.30940108$

$\frac{S_1}{S} = \frac{3}{4}\sqrt{\frac{3}{2}} = .918558654$

HEXAKIS OCTAHEDRON

$\tan\theta_D = \sqrt{6}\sqrt{2+\sqrt{2}}$, $\theta_D = 77.5410898$

$\sin\frac{\alpha_1}{2} = \frac{1}{2\sqrt{3\sqrt{2}}}\left(\sqrt{\sqrt{2}-1} + \sqrt{(\sqrt{2}+1)2}\right)$, $\alpha_1 = 87.20196377$

$\sin\frac{\alpha_2}{2} = \frac{\sqrt{\sqrt{2}+1}}{2\sqrt{2\sqrt{2}}}$, $\alpha_2 = 55.02469615$

$\alpha_3 = 180 - \alpha_1 - \alpha_2 = 37.77334008$

$\frac{2i_D}{S_1} = \frac{\sqrt{6}\sqrt{2+\sqrt{2}}}{2\sqrt{13+6\sqrt{2}}}\left(\frac{5+3\sqrt{2}}{1+\sqrt{2}}\right) = 1.869135702$

$\frac{2P}{S_1} = \frac{1}{2}\frac{5+3\sqrt{3}}{1+\sqrt{3}} = 1.91421356$

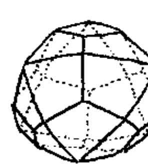
$\frac{2N_4}{S_1} = \sqrt{\frac{3}{\sqrt{2}}}\frac{5+3\sqrt{2}}{(1+\sqrt{2})\left(\sqrt{\sqrt{2}-1} + \sqrt{2(\sqrt{2}+1)}\right)} = 1.96271850$

$\frac{2N_6}{S_1} = \frac{5+3\sqrt{2}}{\sqrt{\sqrt{2}}(1+\sqrt{2})^{3/2}} = 2.07192983$

$\frac{2N_8}{S_1} = \sqrt{\frac{3}{\sqrt{2}}}\frac{5+3\sqrt{2}}{(1+\sqrt{2})\left(\sqrt{\sqrt{2}-1}\cdot 2 - \sqrt{\sqrt{2}+1}\right)} = 2.26303344$

$\frac{S_1}{S} = \frac{2\sqrt{6}\sqrt{2+\sqrt{2}}(\sqrt{2}+1)}{5+3\sqrt{2}} = 2.36445241$

TRAPEZOIDAL ICOSITETRAHEDRON



$\tan\theta_1 = \sqrt{2}\sqrt{2+\sqrt{2}}$, $\theta_D = 69.0589795$

$\sin\frac{\alpha_2}{2} = \frac{1}{2}\sqrt{1+\frac{1}{\sqrt{2}}}$, $\alpha_2 = 81.57894188$

$\sin\frac{\alpha_1}{2} = \frac{1}{2\sqrt{2\sqrt{2}}}\left(\sqrt{\sqrt{2}-1} + \sqrt{2(1+\sqrt{2})}\right)$, $\alpha_1 = 115.2631744$

$\frac{2i_D}{S_1} = \frac{(2+\sqrt{2})^{3/2}}{\sqrt{5+2\sqrt{2}}} = 2.25475194$

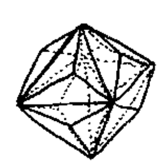
$\frac{2P}{S_1} = 1+\sqrt{2} = 2.41421356$

$\frac{2N_3}{S_1} = \frac{\sqrt{6\sqrt{2}} + 2\sqrt{3\sqrt{2}}}{\sqrt{\sqrt{2}-1} + \sqrt{2(\sqrt{2}+1)}} = 2.47538817$

$\frac{2N_4}{S_1} = \sqrt{2(2+\sqrt{2})} = 2.61312593$

$\frac{S_1}{S} = \frac{\sqrt{2}\sqrt{2+\sqrt{2}}}{1+\sqrt{2}} = 1.0823922$

TRIAKIS OCTAHEDRON



$\tan\theta_D = 2+\sqrt{2}$, $\theta_D = 73.67505006$

$\sin\frac{\alpha_1}{2} = \frac{1}{2}\sqrt{1-\frac{1}{\sqrt{2}}}$, $\alpha_1 = 31.3997148$

$\sin\frac{\alpha_2}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$, $\alpha_2 = 117.2005704$

$\frac{2i_D}{S_1} = \frac{2+\sqrt{2}}{\sqrt{7+4\sqrt{2}}} = .95968298$

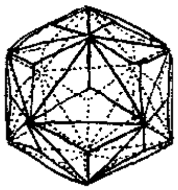
$\frac{2P}{S_1} = 1$

$\frac{2N_3}{S_1} = \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} = 1.01461187$

$\frac{2N_8}{S_1} = \sqrt{2} = 1.41421356$

$\frac{S_1}{S} = 2+\sqrt{2} = 3.41421356$

TRIAKIS ICOSAHEDRON



$TAN\,\theta_D = 3\tau+1$, $\theta_D = 80.3062761$

$\sin\frac{\alpha_1}{2} = \frac{1}{10}(\tau+7)$, $\alpha_1 = 119.0393509$

$\sin\frac{\alpha_2}{2} = \frac{1}{2\sqrt{\tau+2}}$, $\alpha_2 = 30.4803246$

$\frac{2i_D}{S_1} = \frac{4\tau+3}{\sqrt{15\tau+11}} = 1.59493157$

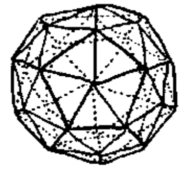
$\frac{2P}{S_1} = \tau = 1.61803399$

$\frac{2N_3}{S_1} = \frac{\sqrt{3}(4\tau+3)}{\sqrt{45\tau+29}} = 1.625960794$

$\frac{2N_{10}}{S_1} = \sqrt{\tau+2} = 1.90211303$

$\frac{S_1}{S} = \tau+2 = 3.61803399$

PENTAKIS DODECAHEDRON



$TAN\,\theta_D = 3\tau$, $\theta_D = 78.35927686$

$\sin\frac{\alpha_1}{2} = \frac{5-\tau}{6}$, $\alpha_1 = 68.61872093$

$\sin\frac{\alpha_2}{2} = \frac{\tau}{2\sqrt{3}}$, $\alpha_2 = 55.69063953$

$\frac{2i_D}{S_1} = \frac{3}{\sqrt{58-35\tau}} = 2.56418649$

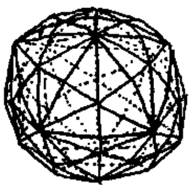
$\frac{2P}{S_1} = \tau^2 = 2.61803399$

$\frac{2N_5}{S_1} = \frac{2}{19}\sqrt{131\tau+87} = 2.730083798$

$\frac{2N_6}{S_1} = \tau\sqrt{3} = 2.80251708$

$\frac{S_1}{S} = \frac{3}{2\tau} = .927050983$

HEXAKIS ICOSAHEDRON



$TAN\,\theta_D = \sqrt{6}\sqrt{4\tau+3}$, $\theta_D = 82.44394595$

$\sin\frac{\alpha_1}{2} = \frac{4\tau+1}{2\sqrt{3}\sqrt{3+4\tau}}$, $\alpha_1 = 88.9918019$

$\sin\frac{\alpha_2}{2} = \frac{1}{2}\sqrt{\frac{1}{10}(3+4\tau)}$, $\alpha_2 = 58.2379196$

$\sin\frac{\alpha_3}{2} = \frac{1}{2\tau}\sqrt{\frac{5}{6}}$, $\alpha_3 = 32.7702785$

$\frac{2i_D}{S_1} = 2\sqrt{6\frac{4\tau+3}{25\tau+19}}\left(\frac{1}{\sqrt{5}}+\sqrt{\frac{7-4\tau}{3+4\tau}}\right)^{-1} = 2.86187618$

$\frac{2P}{S_1} = \frac{2}{\frac{1}{\sqrt{5}}+\sqrt{\frac{7-4\tau}{3+4\tau}}} = 2.92705098$

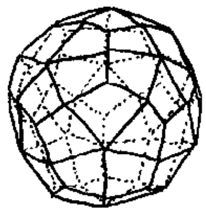
$\frac{2N_4}{S_1} = \frac{2\sqrt{6(3+4\tau)}}{4\tau+1}\left(\frac{1}{\sqrt{5}}+\sqrt{\frac{7-4\tau}{3+4\tau}}\right)^{-1} = 2.9531473$

$\frac{2N_6}{S_1} = \frac{10\sqrt{2}}{\sqrt{3+4\tau}+\sqrt{5(7-4\tau)}} = 3.00750478$

$\frac{2N_{10}}{S_1} = 2\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{5}}+\sqrt{\frac{7-4\tau}{3+4\tau}}\right)^{-1} = 3.20642370$

$\frac{S_1}{S} = \frac{\sqrt{6}}{2}\left(\frac{\sqrt{3+4\tau}}{\sqrt{5}}+\sqrt{7-4\tau}\right) = 2.57554593$

TRAPEZOIDAL HEXECONTAHEDRON



$TAN\,\theta_D = \sqrt{2\sqrt{4\tau+3}}$, $\theta_D = 77.06068156$

$\sin\frac{\alpha_1}{2} = \frac{1}{2}\sqrt{\frac{1}{10}(23+4\tau)}$, $\alpha_1 = 118.2686775$

$\sin\frac{\alpha_2}{2} = \frac{\tau^2}{2\sqrt{\tau+2}}$, $\alpha_2 = 86.9741555$

$\sin\frac{\alpha_3}{2} = \frac{3}{2\tau}\sqrt{\frac{1}{10}(\tau+4)}$, $\alpha_3 = 67.7830116$

$\frac{2i_D}{S_1} = \frac{3}{(3\tau-4)}\sqrt{\frac{2(4\tau+3)}{8\tau+7}} = 3.42327193$

$\frac{2P}{S_1} = \frac{3}{3\tau-4} = 3.51246118$

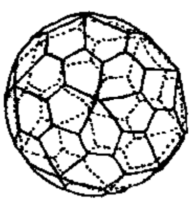
$\frac{2N_3}{S_1} = \frac{3}{3\tau-4}\sqrt{\frac{30}{23+4\tau}} = 3.5437768$

$\frac{2N_4}{S_1} = \frac{3}{\sqrt{5}}\sqrt{2(\tau+2)} = 3.6090057$

$\frac{2N_5}{S_1} = \frac{\sqrt{10}}{3\tau-4} = 3.70245917$

$\frac{S_1}{S} = \frac{1}{3}\sqrt{2(4\tau+3)}\,(3\tau-4) = 1.23916012$

PENTAGONAL HEXECONTAHEDRON



SAY $a = 1.778973359$

$TAN\,\theta_D = \sqrt{\frac{(\tau a)^2+1}{7-4\tau}}$, $\theta_D = 76.5893663$

$\sin\frac{\alpha_1}{2} = \frac{\sqrt{3}}{2}\sqrt{\frac{(\tau a)^2+\frac{4}{3\tau}}{(\tau a)^2+1}}$, $\alpha_1 = 118.1366228$

$\sin\frac{\alpha_2}{2} = \frac{\sin 36}{\sqrt{1+1/(\tau a)^2}}$, $\alpha_2 = 67.4535089$

$\frac{2i_D}{S_1} = \left(\frac{2P}{S_1}\right)\sin\theta_D = 4.0797463$

$\frac{2P}{S_1} = 2\left(\frac{1}{\tau a}+\sqrt{\frac{7-4\tau}{3(\tau a)^2+\frac{4}{\tau}}}\right)^{-1} = 4.1119176$

$\frac{2N_3}{S_1} = \left(\frac{2P}{S_1}\right)\sqrt{\frac{(\tau a)^2+1}{(\tau a)^2+\frac{4}{3\tau}}} = 4.1514398$

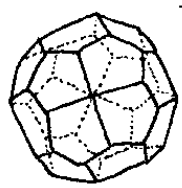
$\frac{2N_5}{S_1} = \left(\frac{2P}{S_1}\right)\sqrt{1+\left(\frac{1}{\tau a}\right)^2} = 4.3529927$

$\frac{S_1}{S} = \frac{TAN\,\theta_D}{(2P/S_1)} = 1.0199883$

PENTAGONAL ICOSITETRAHEDRON

SAY $a = \sqrt{2}\left(b+\sqrt{2b^2-1}\right)$ WHERE

$b^2 = \frac{1}{6}\left(\sqrt[3]{17+3\sqrt{33}}+\sqrt[3]{17-3\sqrt{33}}\right)+\frac{1}{3}$



THEN $TAN\,\theta_D = \sqrt{a^2+1}$, $\theta_D = 68.15461645$

$\sin\frac{\alpha_1}{2} = \frac{1}{2}\sqrt{\frac{3a^2+2}{a^2+1}}$, $\alpha_1 = 114.8120745$

$\sin\frac{\alpha_2}{2} = \frac{a}{\sqrt{2}\sqrt{a^2+1}}$, $\alpha_2 = 80.75170209$

$\frac{2i_D}{S_1} = 2\sqrt{\frac{a^2+1}{a^2+2}}\left(\frac{1}{a}+\frac{1}{\sqrt{3a^2+2}}\right)^{-1} = 2.74812867$

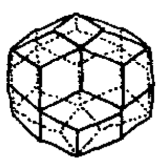
$\frac{2P}{S_1} = 2\left(\frac{1}{a}+\frac{1}{\sqrt{3a^2+2}}\right)^{-1} = 2.96073496$

$\frac{2N_3}{S_1} = 2\left(\frac{1}{a}+\frac{1}{\sqrt{3a^2+2}}\right)^{-1}\sqrt{\frac{a^2+1}{a^2+2/3}} = 3.04337544$

$\frac{2N_4}{S_1} = 2\left(\frac{1}{a}+\frac{1}{\sqrt{3a^2+2}}\right)^{-1}\frac{\sqrt{a^2+1}}{a} = 3.23179904$

$\frac{S_1}{S} = \frac{\sqrt{a^2+1}}{2}\left(\frac{1}{a}+\frac{1}{\sqrt{3a^2+2}}\right) = .842509163$

RHOMBIC TRIACONTAHEDRON



$\theta_D = 72$

$TAN\,\alpha_1 = -2$, $\alpha_1 = 116.565051 2$

$TAN\,\alpha_2 = 2$, $\alpha_2 = 63.4349488$

$\frac{2i_D}{S_1} = \frac{2}{5}(\tau+2)^{3/2} = 2.75276384$

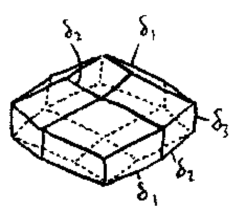
$\frac{2P}{S_1} = \frac{4}{5}(\tau+2) = 2.89442719$

$\frac{2N_3}{S_1} = 2\sqrt{\frac{3}{5}}\sqrt{\tau+2} = 2.94674084$

$\frac{2N_5}{S_1} = 2\tau = 3.236067978$

$\frac{S_1}{S} = \frac{5}{8\sin 36} = 1.06331351$

DERIVATIVE RHOMBOHEDRA



$\delta_1 = 144$

$\delta_2 = 144$

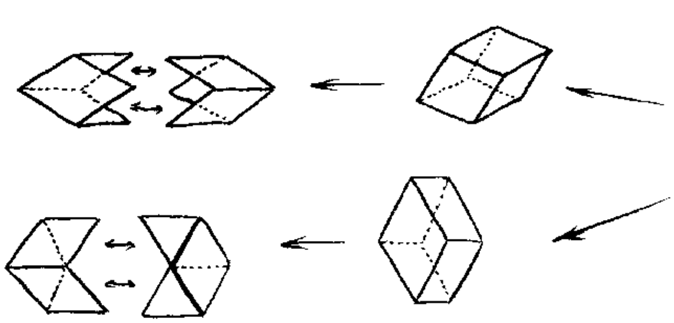
$\delta_3 = 108$

ANGLES θ FROM THE FACE TO THE CENTER DIVIDE δ_1 AND δ_3 IN HALF AND δ_2 INTO

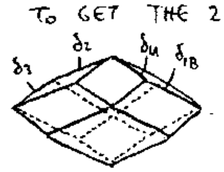
$\cos\theta_{2\text{ TOWARD }1} = \frac{2+\tau}{2\sqrt{5(4-\tau)}}$, $\theta_{2\text{-}1} = 58.38617756$

$\cos\theta_{2\text{ TOWARD }3} = \frac{7-4\tau}{2\sqrt{5(4-\tau)}}$, $\theta_{2\text{-}3} = 85.61382244$

AS MENTIONED EARLIER, THE RHOMBIC TRIACONTAHEDRON IS MADE OF 20 RHOMBIC PARALLELEPIPEDS. THESE CAN BE RECOVERED BY SQUASHING THE TRIACONTAHEDRON TO GET



THE RHOMBIC ICOSAHEDRON, WHICH CAN BE SQUASHED TO GET THE RHOMBIC DODECAHEDRON, WHICH CAN BE SQUASHED 2 WAYS TO GET THE 2 INITIAL PARALLELEPIPEDS.



$\delta_1 = 144$

$\delta_2 = 108$

$\delta_3 = 72$

$TAN\,\theta_{1A\cdot 1A} = \frac{1}{\tau}\sqrt{\tau+2}$, $\theta_{1A\cdot 1A} = 49.61382244$

$TAN\,\theta_{1A\cdot 2} = \frac{\sqrt{\tau+2}}{3\tau-5}$, $\theta_{1A\cdot 2} = 94.38617756$

$\cos\theta_{2\cdot 1} = \frac{3}{2\sqrt{\tau+3}}$, $\theta_{2\cdot 1} = 45.73230145$

$TAN\,\theta_{2\cdot 2} = \sqrt{\tau+2}$, $\theta_{2\cdot 2} = 62.26769855$

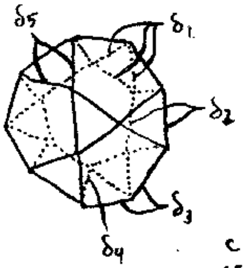
δ_{1B} AND δ_3 ARE DIVIDED IN HALF BY THE CENTER

THIS FITS IN A RECTANGLE OF DIMENSIONS PROPORTIONAL TO $1\times\tau\times\tau^2$

IRREGULAR REGULAR FACED SOLIDS

THE ANGLES θ FOR THESE SHAPES ARE MEASURED FROM A FACE TO THE CENTROID. WHERE THERE IS NO OBVIOUS SYMMETRIC CENTER, c REPRESENTS THE DISTANCE FROM THE INDICATED FACE TO THE CENTROID.

TRIANGULAR HEBESPHENOROTUNDA



$\delta_5 = \delta_{\text{ICOSIDOD.}} = 142.6226319$

$\delta_4 = 90 + \varepsilon = 159.09484255$

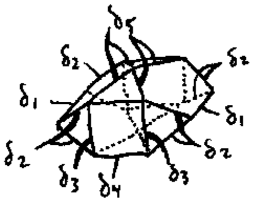
$\delta_3 = 90 - \theta + \varepsilon = 100.812317$

$\delta_2 = 180 - \varepsilon = 110.905157{4}$

$\delta_1 = 2\varepsilon = 138.1896851$

c FROM O:
.6592802

BILUNABIROTUNDA



$\delta_1 = 180 - 2\theta = 63.434949$

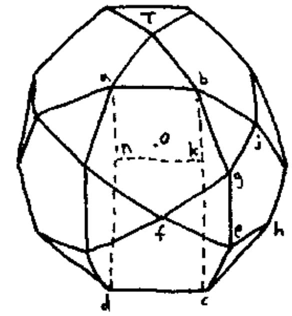
$\delta_2 = 90 - \theta + \varepsilon = 100.812317$

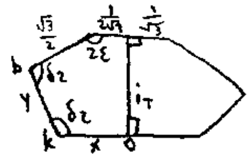
$\delta_3 = 90 + \varepsilon = 159.094843$

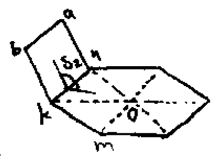
$\delta_4 = 180 - \varepsilon = 110.905157$

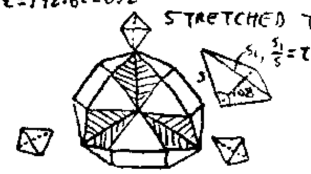
$\delta_5 = 270 - \theta - \varepsilon = 142.622632$

CONSIDER NOW AN ICOSIDODECAHEDRON OF EDGE LENGTH 1 WITH CENTER O AND PLANE O PARALLEL TO T. SAY $\theta = \frac{1}{2}\delta_{\text{DODECAHEDRON}}$ AND $\varepsilon = \frac{1}{2}\delta_{\text{ICOSAHEDRON}}$. ALSO SAY $\delta_5 = \delta_{\text{ICOSIDODECAHEDRON}}$.
WE HAVE $540 = 180 + 2\delta_2 + 2\varepsilon$ SO $\delta_2 = 180 - \varepsilon$.
ALSO, $y = \dfrac{i\tau - \frac{\sqrt{3}}{2}\sin 2\varepsilon}{\cos\delta_2} = 1$ AND $x = \frac{1}{2\sqrt{3}} + \frac{\sqrt{3}}{2}\cos 2\varepsilon - \sin\delta_2 = \frac{\sqrt{3}}{2}$
SO RECT. abcd GETS CUT INTO SQUARE abkm WHICH TOUCHES A HEXAGON CENTERED AT O IN PLANE O. RECOGNIZING kb||eg, kj||fg, km||eh, ONE IS STARTLED BY THE REGULAR FACED, HUT-SHAPED TRIANGULAR HEBESPHENOROTUNDA, AND BECAUSE THE HEXAGON IS MADE OF SIX TRIANGLES, ONE IS FURTHER STARTLED BY THE BILUNAROTUNDA, OF WHICH THREE MUTUALLY INTERSECT IN THE FORMER, LEAVING GAPS FILLED BY THREE STRETCHED TETRAHEDRA.





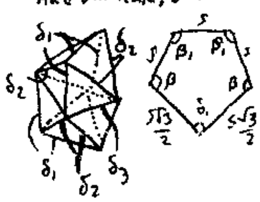




PLANE OF TRIANGLE C FORMS A TETRAHEDRON WITH PLANES OF TRIANGLES A OR B

SNUB DISPHENOID

THIS ONE IS THE MOST DIRECT APPLICATION OF THE EQUATIONS FROM THE PREVIOUS PAGE. THE OTHERS HERE ARE SIMILAR, BUT MORE COMPLICATED



$\tan\frac{\beta_1}{2} = \sqrt{3}\cos\frac{\delta_1}{2}$

$\sin\frac{\beta_1}{2} = \frac{\sqrt{3}}{2}\sin\frac{\delta_2}{2}$

$\frac{1}{\sqrt{3}}\sin\beta - \cos\frac{\delta_1}{2}\cos\beta = \sin\delta_2$

$2\sqrt{3}\cos\beta - 3\cos\delta_3 = 1$

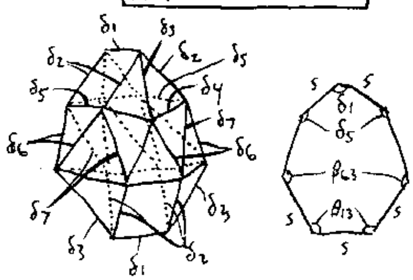
$2\beta_1 + 2\beta + \delta_1 = 540$

$h = 1.5678618$

$\beta_1 = 98.31320612$

$\beta = 123.5876294$

DISPHENOCINGULUM

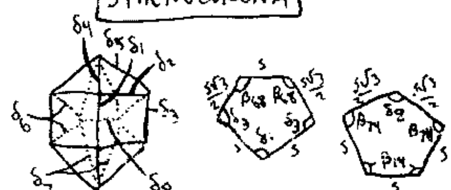


$h = 2.2088759$

$\beta_{13} = 128.791130$

$\beta_{63} = 116.693081$

SPHENOCORONA



c, FROM MID δ_1 EDGE:
.6638236

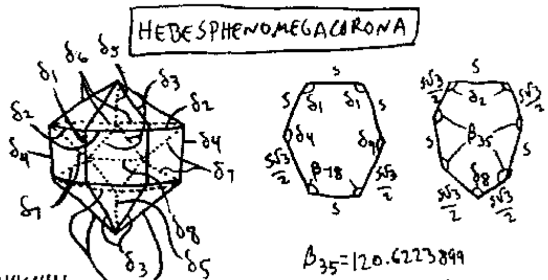
$h = 1.3132954$

$\beta_{14} = 106.82364$

$\beta_{68} = 114.03496$

$\beta_{74} = 97.45552$

HEBESPHENOMEGACORONA



$\beta_{35} = 120.6223899$

$\beta_{78} = 104.500641$

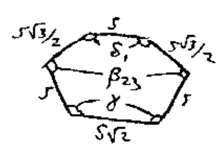
$h = 1.8146441$

c, FROM MID CENTER SQUARE:
.8456766

SNUB SQUARE ANTIPRISM

THIS SHAPE IS OBTAINED BY SQUASHING THE SNUB CUBE, ELIMINATING 4 SQUARES AND 8 TRIANGLES

THE DIHEDRAL ANGLES MUST BE DISTORTED TO ACHIEVE THIS FEAT.



$\beta_{23} = 94.1550527$

$\gamma = 120.404318$

THIS SITUATION CONJURES THE FOLLOWING ADDITIONAL EQUATIONS

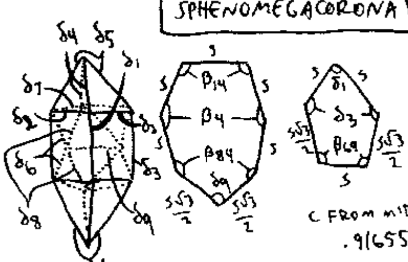
4 TRIANGLES MEET 1 SQUARE

$$\boxed{\sqrt{6}\sin\frac{\delta_3}{2} + \sqrt{3}\cos\delta_1 = 1}$$

$$\boxed{\frac{1}{\sqrt{3}}\sin\gamma - \cos\frac{\delta_3}{2}\cos\gamma = \sin\delta_1}$$

$$\boxed{2\sqrt{2}\cos\gamma - 3\cos\delta_2 = 2}$$

SPHENOMEGACORONA



$\beta_4 = 103.0910137$

$\beta_{14} = 141.5455069$

$\beta_{69} = 96.27341834$

$\beta_{84} = 124.6220547$

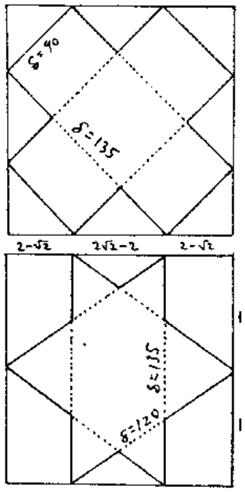
c FROM MID δ_1 EDGE:
.9165579

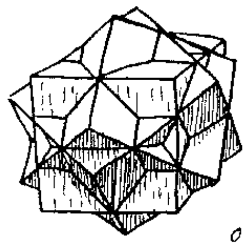
$h = 1.6648364$

COMPOUNDS

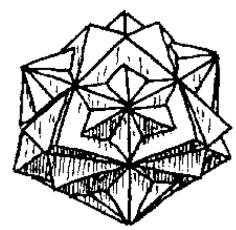
THERE ARE MANY WAYS TO MAKE COMPOUNDS. THE PRIMARY WAY ON THIS PAGE IS TO TAKE A CUBE, THEN ROTATE COPIES OF ITSELF AROUND ITS VARIOUS SYMMETRY AXES INTO A DIFFERENT POSITION. ROTATION AROUND THE 3 AXES THROUGH THE CENTER OF A CUBE'S FACES BY 45°, AND REMOVING THE ORIGINAL CUBE GIVES 3 CUBES, AT LEFT. ROTATING 60° AROUND THE FOUR AXES CONNECTING OPPOSITE CORNERS GIVES 5 AND 4 CUBES, AND 90° AROUND THE 6 AXES THROUGH OPPOSITE EDGES GIVES 7 AND 6 CUBES. SOLIDS EMBRACE EACH OTHER IN MANY WAYS SO THESE BASIC CUBE ARRANGEMENTS MERELY SIT AT THE THRESHHOLD OF AN ENORMOUS VARIETY OF INTERESTING COMPOUNDS.

THREE CUBES

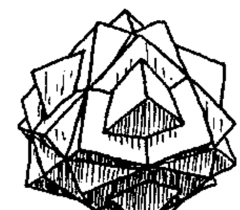




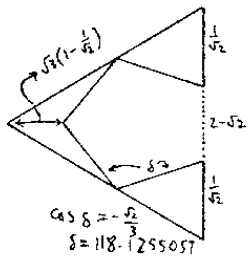
FIVE CUBES



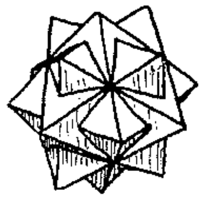
FOUR CUBES



THREE OCTAHEDRA



$\cos\delta = -\frac{\sqrt{2}}{3}$
$\delta = 118.1255057$

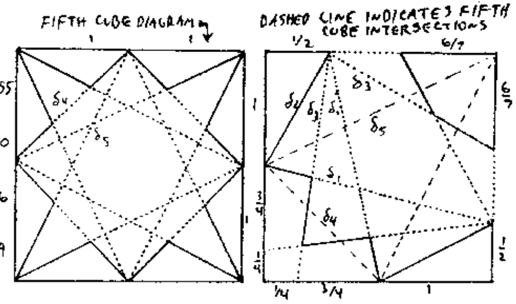


THE δ ON THIS PAGE ARE MEASURED ON THE OUTSIDE.

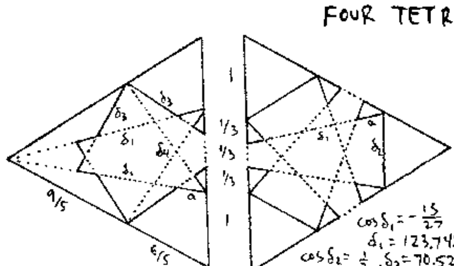
FIFTH CUBE DIAGRAM

DASHED LINE INDICATES FIFTH CUBE INTERSECTIONS

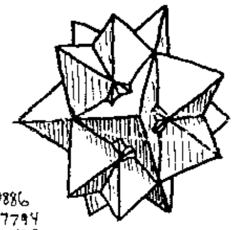
$\cos\delta_1 = -\frac{8}{9}$
$\delta_1 = 152.7339555$
$\cos\delta_2 = -\frac{4}{9}$
$\delta_3 = \delta_2 = 116.3878600$
$\cos\delta_4 = -\frac{1}{3}$
$\delta_4 = 109.4712206$
$\cos\delta_5 = -\frac{2}{3}$
$\delta_5 = 131.8103149$



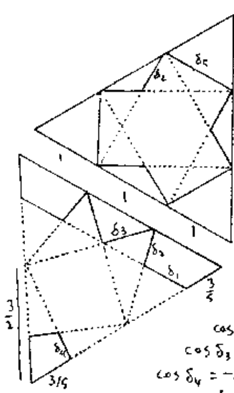
FOUR TETRAHEDRA

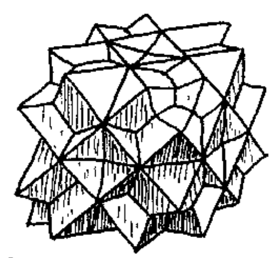


$\cos\delta_1 = -\frac{15}{27}$, $\delta_1 = 123.7489886$
$\cos\delta_2 = \frac{1}{3}$, $\delta_2 = 70.5287794$
$\cos\delta_3 = \frac{1}{27}$, $\delta_3 = 87.8774486$
$\cos\delta_4 = -\frac{1}{27}$, $\delta_4 = 92.1225514$



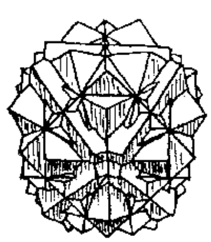
FOUR OCTAHEDRA



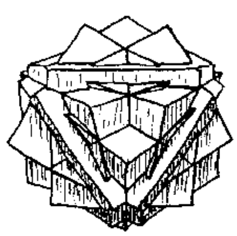


$\cos\delta_1 = -\frac{23}{27}$, $\delta_1 = 148.4136619$
$\cos\delta_2 = -\frac{5}{9}$, $\delta_2 = 123.7489886$
$\cos\delta_3 = -\frac{25}{27}$, $\delta_3 = 157.8083934$
$\cos\delta_4 = -\frac{11}{27}$, $\delta_4 = 114.0420759$
$\cos\delta_5 = -\frac{1}{3}$, $\delta_5 = 109.4712206$

SEVEN CUBES



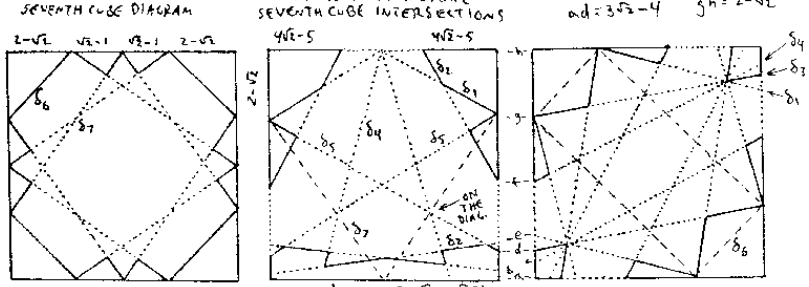
SIX CUBES



$\cos\delta_1 = \frac{2-\sqrt{2}}{4}$, $\delta_1 = 81.5789419$
$\cos\delta_2 = -\frac{1}{4}$, $\delta_2 = 104.4775122$
$\delta_3 = 120$
$\cos\delta_4 = -\frac{2+\sqrt{2}}{4}$, $\delta_4 = 148.6007852$
$\cos\delta_5 = -\frac{2\sqrt{2}+1}{4}$, $\delta_5 = 163.1578838$
$\delta_6 = 90$
$\delta_7 = 120$
$ab = \frac{1}{7}(16-11\sqrt{2})$ $ae = \frac{1}{3}(4-\sqrt{2})$
$ac = \frac{1}{7}(8-5\sqrt{2})$ $fg = \frac{1}{3}(3\sqrt{2}-2)$
$ad = 3\sqrt{2}-4$ $gh = 2-\sqrt{2}$

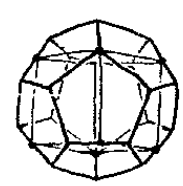
SEVENTH CUBE DIAGRAM

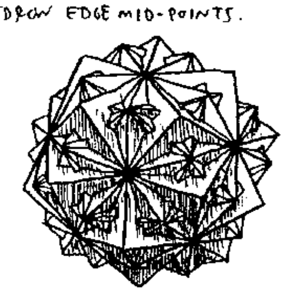
DASHED LINES INDICATE SEVENTH CUBE INTERSECTIONS



10 PSUEDO-CUBES

THIS CONFIGURATION OF 10 ALMOST CUBES WAS NOTED BY GEORGE ODOM. EACH HAS 2 VERTICES THAT SIT ON DODECAHEDRON VERTICES, AND 6 THAT SIT ON DODECAHEDRON EDGE MID-POINTS.

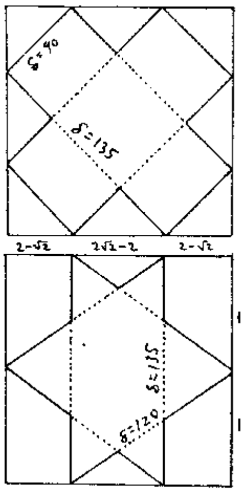


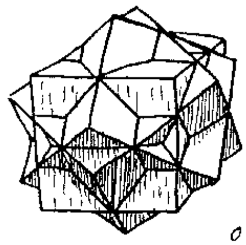


COMPOUNDS

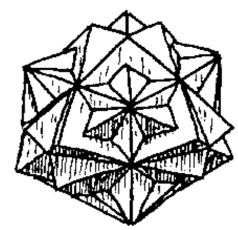
THERE ARE MANY WAYS TO MAKE COMPOUNDS. THE PRIMARY WAY ON THIS PAGE IS TO TAKE A CUBE, THEN ROTATE COPIES OF ITSELF AROUND ITS VARIOUS SYMMETRY AXES INTO A DIFFERENT POSITION. ROTATION AROUND THE 3 AXES THROUGH THE CENTER OF A CUBE'S FACES BY 45°, AND REMOVING THE ORIGINAL CUBE GIVES 3 CUBES, AT LEFT. ROTATING 60° AROUND THE FOUR AXES CONNECTING OPPOSITE CORNERS GIVES 5 AND 4 CUBES, AND 90° AROUND THE 6 AXES THROUGH OPPOSITE EDGES GIVES 7 AND 6 CUBES. SOLIDS EMBRACE EACH OTHER IN MANY WAYS SO THESE BASIC CUBE ARRANGEMENTS MERELY SIT AT THE THRESHHOLD OF AN ENORMOUS VARIETY OF INTERESTING COMPOUNDS.

THREE CUBES

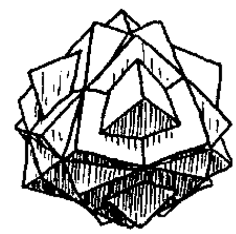




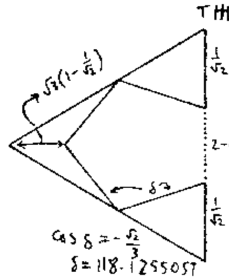
FIVE CUBES



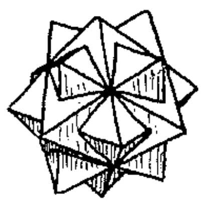
FOUR CUBES



THREE OCTAHEDRA



$\cos\delta=-\frac{\sqrt{2}}{3}$
$\delta=118.1255057$

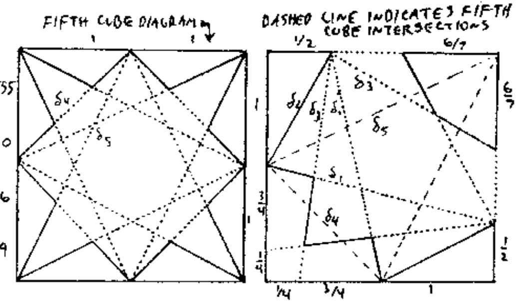


THE δ ON THIS PAGE ARE MEASURED ON THE OUTSIDE.

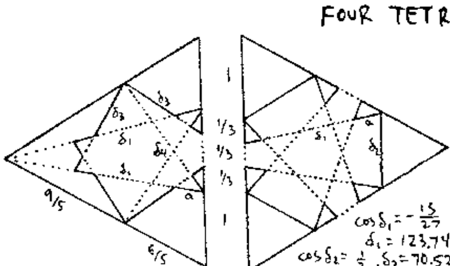
$\cos\delta_1=-\frac{8}{9}$
$\delta_1=152.7339555$
$\cos\delta_2=-\frac{4}{9}$
$\delta_3=\delta_2=116.3878600$
$\cos\delta_4=-\frac{1}{3}$
$\delta_4=109.4712206$
$\cos\delta_5=-\frac{2}{3}$
$\delta_5=131.8103149$

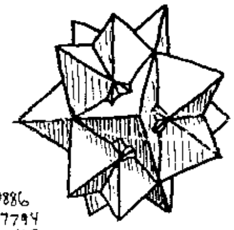
FIFTH CUBE DIAGRAM

DASHED LINE INDICATES FIFTH CUBE INTERSECTIONS



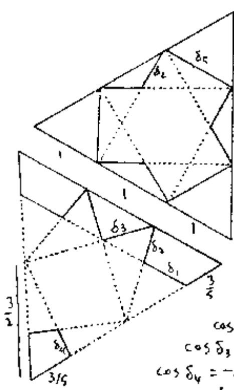
FOUR TETRAHEDRA

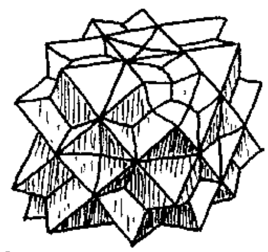




$\cos\delta_1=-\frac{15}{27}$
$\delta_1=123.7489886$
$\cos\delta_2=\frac{1}{3}$, $\delta_2=70.5287794$
$\cos\delta_3=\frac{1}{27}$, $\delta_3=87.8774486$
$\cos\delta_4=-\frac{1}{27}$, $\delta_4=92.1225514$

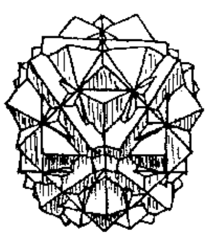
FOUR OCTAHEDRA



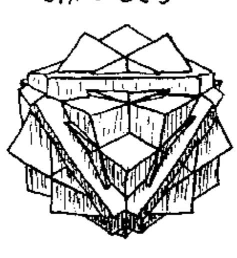


$\cos\delta_1=-\frac{23}{27}$, $\delta_1=148.4136619$
$\cos\delta_2=-\frac{5}{9}$, $\delta_2=123.7489886$
$\cos\delta_3=-\frac{25}{27}$, $\delta_3=157.8083934$
$\cos\delta_4=-\frac{11}{27}$, $\delta_4=114.0420759$
$\cos\delta_5=-\frac{1}{3}$, $\delta_5=109.4712206$

SEVEN CUBES



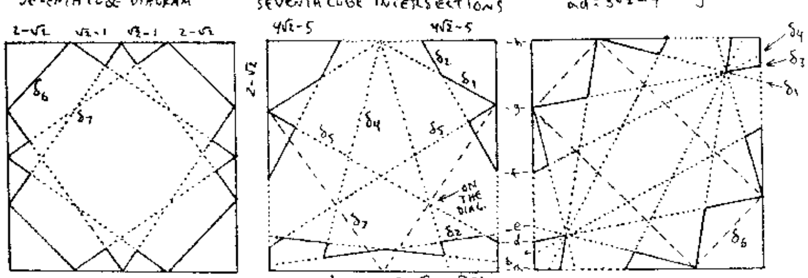
SIX CUBES



$\cos\delta_1=\frac{2-\sqrt{2}}{4}$, $\delta_1=81.5789419$
$\cos\delta_2=-\frac{1}{4}$, $\delta_2=104.4775122$
$\delta_3=120$
$\cos\delta_4=-\frac{2+\sqrt{2}}{4}$, $\delta_4=148.6007852$
$\cos\delta_5=-\frac{2\sqrt{2}+1}{4}$, $\delta_5=163.1578838$
$\delta_6=90$
$\delta_7=120$
$ab=\frac{1}{7}(16-11\sqrt{2})$ $ae=\frac{1}{7}(4-\sqrt{2})$
$ac=\frac{1}{7}(8-5\sqrt{2})$ $fg=\frac{1}{7}(3\sqrt{2}-2)$
$ad=3\sqrt{2}-4$ $gh=2-\sqrt{2}$

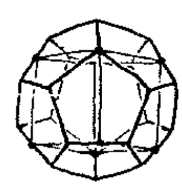
SEVENTH CUBE DIAGRAM

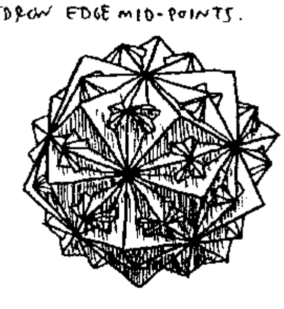
DASHED LINES INDICATE SEVENTH CUBE INTERSECTIONS



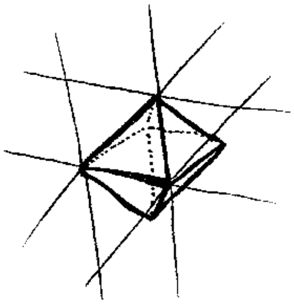
10 PSUEDO-CUBES

THIS CONFIGURATION OF 10 ALMOST CUBES WAS NOTED BY GEORGE ODOM. EACH HAS 2 VERTICES THAT SIT ON DODECAHEDRON VERTICES, AND 6 THAT SIT ON DODECAHEDRON EDGE MID-POINTS.

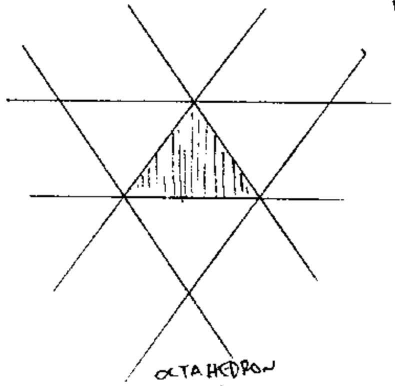




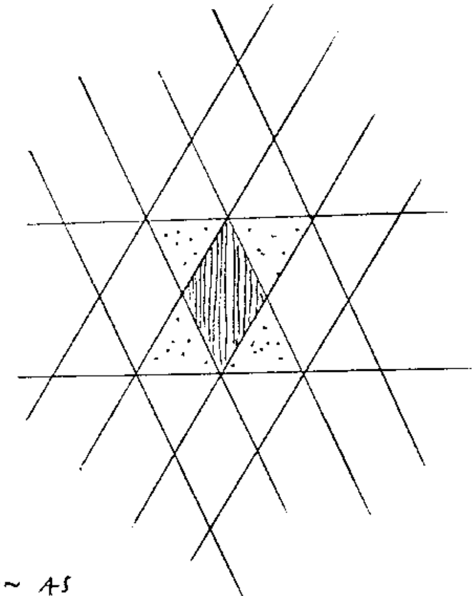
STELLATIONS



TO STELLATE A SOLID IS TO EXTEND THE FACES IN THEIR OWN PLANES, THE EDGES OF THE STELLATION BEING DEFINED BY THE VARIOUS INTERSECTIONS OF THE EXTENDED FACES. THE DIAGRAMS SHOW A PLANE OF A FACE WITH ALL THOSE INTERSECTIONS. FILLING IN VARIOUS PATTERNS ON THE DIAGRAMS WILL INSTANTLY TRANSFORM YOUR CUTE LITTLE BALL INTO A POINTED EXTRAVAGANZA. FOR INSTANCE, FILLING THE TRIANGLES OF THE OCTAHEDRON'S DIAGRAM AT LEFT WILL FORGE THE STELLA OCTANGULA, WHICH CAN ALSO BE SEEN AS BEING TWO INTERSECTING TETRAHEDRA. INCIDENTALLY, THE STELLA OCTANGULA TOGETHER WITH THE OCTAHEDRON CAN BE STACKED LIKE BRICKS TO FILL SPACE WITH NO GAPS.



OCTAHEDRON STELLATION DIAGRAM

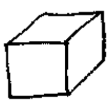


RHOMBIC DODECAHEDRON

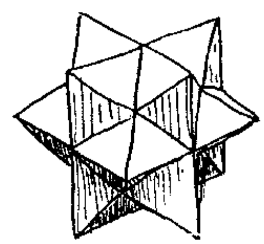
THE DOTTED TRIANGLES COMPRISE THE RHOMBIC DODECAHEDRON STELLATION LISTED IN THE TABLE, AND SHOWN BELOW

ONCE A DIAGRAM IS FOUND AND YOU ARE STILL NOT DISCOURAGED, THE DIHEDRAL ANGLES ARE NO MORE THAN ANGLES MORE EASILY FOUND BY EXAMINING FACES IN THE ORIGINAL SOLID. WE ARE NOT MISLED BY ANY AMOUNT OF STELLATION ARTIFICE. HOWEVER, THE EXTERIOR FAÇADE CAN BE A SUBJECT OF STUDY IN ITS OWN RIGHT WITH THOSE PROPERLY DISPOSED.

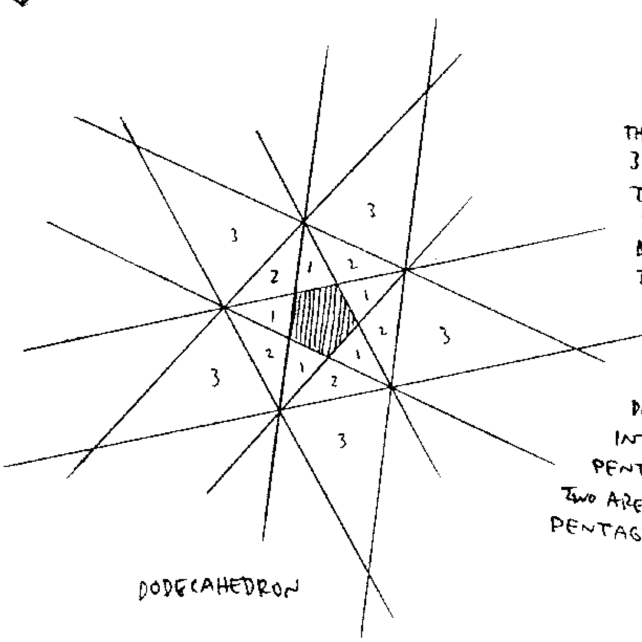
THE OCTAHEDRON HAS ONLY ONE STELLATION



THE CUBE HAS NO CLOSED STELLATIONS.

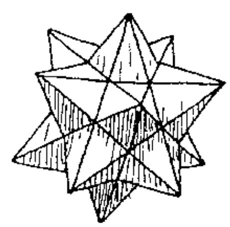


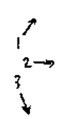
THE TETRAHEDRON HAS NO STELLATIONS

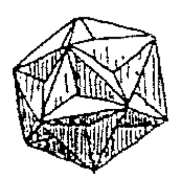


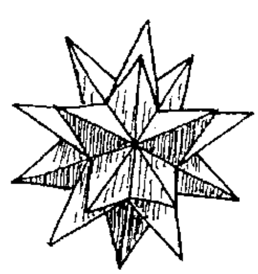
DODECAHEDRON

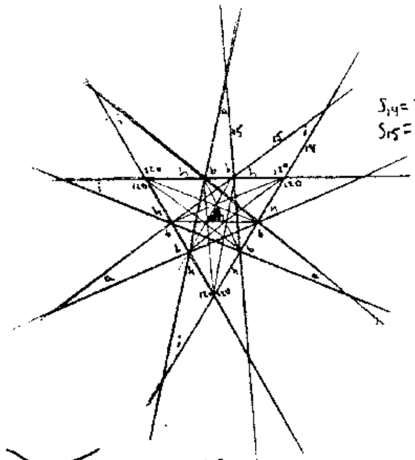
THE DODECAHEDRON HAS 3 STELLATIONS. FILLING THE SHAPE #1 GIVES THE SMALL STELLATED DODECAHEDRON, 2 GIVES THE GREAT DODECAHEDRON, 3 GIVES THE GREAT STELLATED DODECAHEDRON. THEREFORE, THE GREAT DODECAHEDRON IS 12 INTERPENETRATING PENTAGONS, AND THE OTHER TWO ARE 12 INTERPENETRATING PENTAGRAMS.







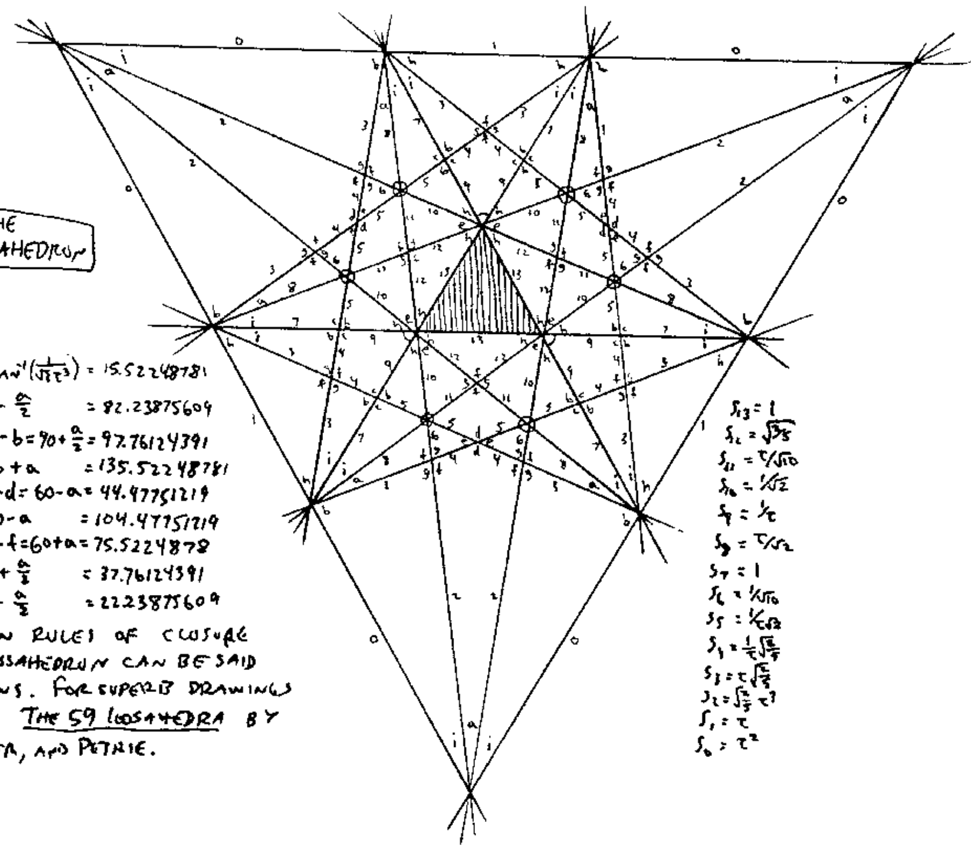




$S_{14} = \tau^3$
$S_{15} = \tau^2\sqrt{2}$

JOINING SIDES CAN ONLY HAVE $\delta = 70.52877937$ OR $\delta = 41.8103149$

THE ICOSAHEDRON

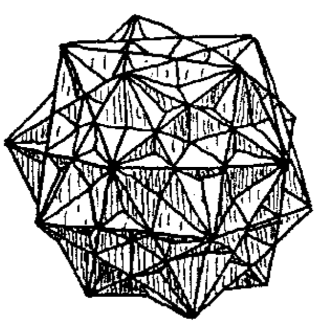


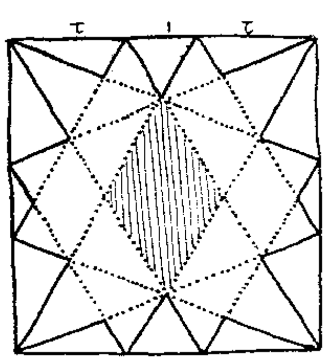
$a = 2\tan^{-1}\left(\frac{1}{\sqrt{3}\tau^3}\right) = 15.52248781$
$b = 90 - \frac{a}{2} = 82.23875609$
$c = 180 - b = 90 + \frac{a}{2} = 97.76124391$
$d = 120 + a = 135.52248781$
$e = 180 - d = 60 - a = 44.47751219$
$f = 120 - a = 104.47751219$
$g = 180 - f = 60 + a = 75.5224878$
$h = 30 + \frac{a}{2} = 37.76124391$
$i = 30 - \frac{a}{2} = 22.23875609$

BY IMPOSING CERTAIN RULES OF CLOSURE AND SYMMETRY, THE ICOSAHEDRON CAN BE SAID TO HAVE 59 STELLATIONS. FOR SUPERB DRAWINGS AND RELATED INFO, SEE THE 59 ICOSAHEDRA BY COXETER, DUVAL, FLATHER, AND PETRIE.

$S_{13} = 1$
$S_{12} = \sqrt{3/5}$
$S_{11} = \tau/\sqrt{10}$
$S_{10} = 1/\sqrt{2}$
$S_9 = 1/\tau$
$S_8 = \tau/\sqrt{2}$
$S_7 = 1$
$S_6 = 1/\sqrt{10}$
$S_5 = 1/\tau\sqrt{2}$
$S_4 = \frac{1}{\tau}\sqrt{\frac{3}{5}}$
$S_3 = \tau\sqrt{\frac{\tau}{5}}$
$S_2 = \sqrt{\frac{2}{5}}\,\tau^2$
$S_1 = \tau$
$S_0 = \tau^2$

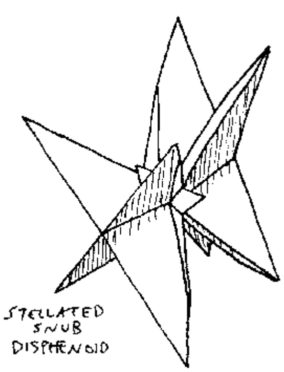
OFTEN, STELLATIONS ARE COMPOUNDS OF SIMPLER STELLATIONS OR OTHER SHAPES. A STRIKING EXAMPLE OF THIS IS THE COMPOUND OF FIVE CUBES WHICH IN THIS CASE IS A STELLATION OF THE RHOMBIC TRIACONTAHEDRON. THE SQUARE'S EDGES ARE DIVIDED IN THE GOLDEN SECTION.





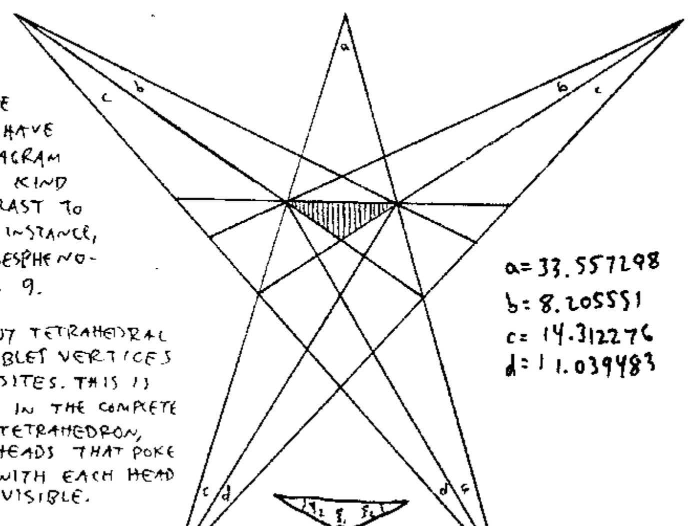
ONE GOOD THING ABOUT THE DUALS LISTED IN THE TABLE IS THAT THEY HAVE ONLY ONE STELLATION DIAGRAM SINCE THERE IS ONLY ONE KIND OF FACE. THIS IS IN CONTRAST TO THE SNUB DODECAHEDRON, FOR INSTANCE, WHICH NEEDS 3, OR THE HEBESPHENOMEGACORONA, WHICH NEEDS 9.

AN INTERESTING THING ABOUT TETRAHEDRAL SYMMETRY IS THAT IT ENABLES VERTICES THAT ARE NOT POLAR OPPOSITES. THIS IS DRAMATICALLY POINTED OUT IN THE COMPLETE STELLATION OF THE TRIAKIS TETRAHEDRON, WHICH HAS FOUR ARROW HEADS THAT POKE THROUGH EACH OTHER, WITH EACH HEAD AND TAIL VISIBLE.



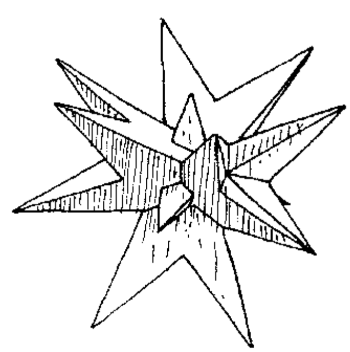
STELLATED SNUB DISPHENOID

TRIAKIS TETRAHEDRON



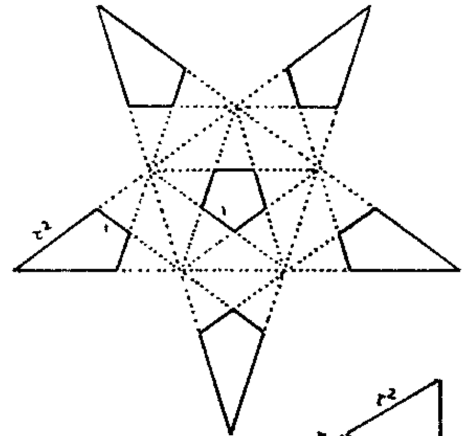
$a = 33.557298$
$b = 8.205551$
$c = 14.312276$
$d = 11.039483$

$\xi_1 = 112.885381$
$\xi_2 = 33.557310$

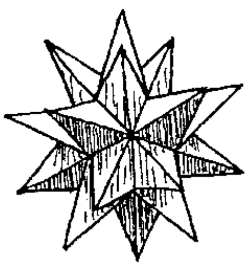


Uniform Polyhedra

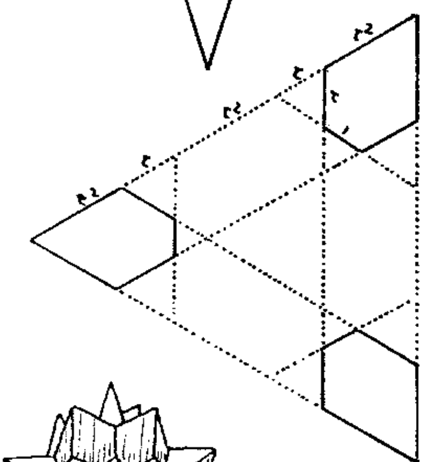
THE GREAT ICOSIDODECAHEDRON



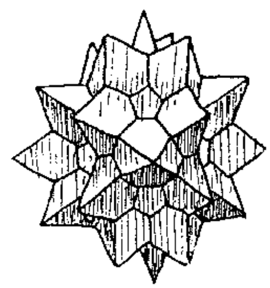
A UNIFORM POLYHEDRON HAS FACES THAT ARE ALL REGULAR POLYGONS AND VERTICES THAT ARE ALL ALIKE. ASIDE FROM THE PRISMS, ANTIPRISMS, PLATONIC, AND ARCHIMEDEAN SOLIDS, THERE ARE 57 OTHER UNIFORM POLYHEDRA, OR 58 IF YOU ALLOW 4 FACES TO MEET AT THE SAME EDGE. IF ALL THE FACES ARE THE SAME POLYGON, THE SOLID IS ALSO CALLED REGULAR. THERE ARE THEN 9 REGULAR POLYHEDRA: THE PLATONIC SOLIDS, AND THE 4 SO CALLED KEPLER-POINSOT SOLIDS: THE GREAT DODECAHEDRON, GREAT ICOSAHEDRON, GREAT AND SMALL STELLATED DODECAHEDRA.

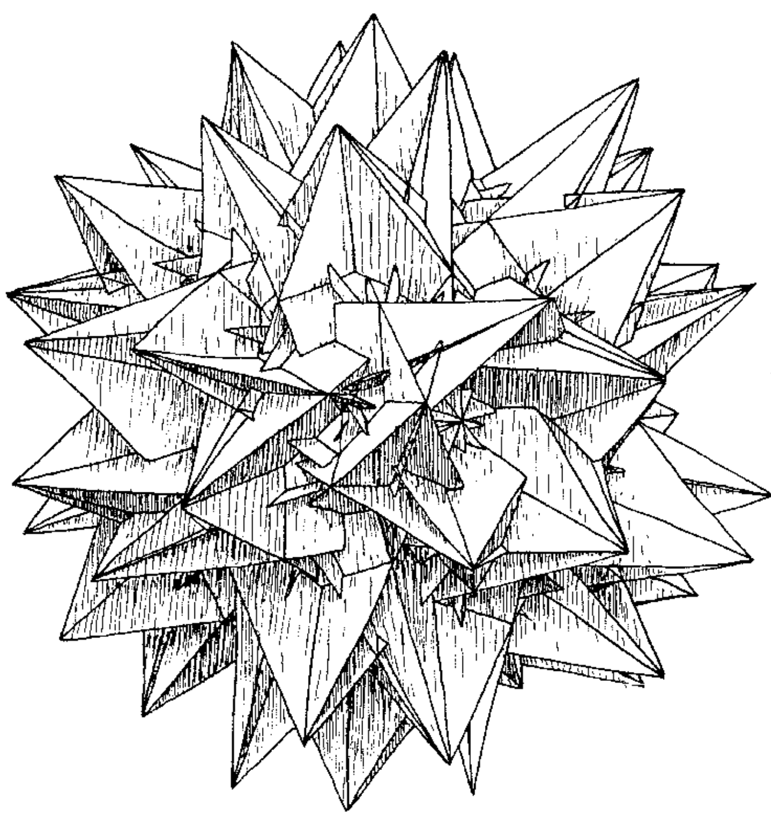


GREAT STELLATED DECAHEDRON



IF THIS PAGE IN PARTICULAR INTERESTS YOU, THEN GET DOWN TO THE LIBRARY AND REVEL IN THE SPLENDOR OF THE INTERLIBRARY LOAN BY ORDERING A COPY OF THE ARTICLE "UNIFORM POLYHEDRA" BY COXETER, LONGUET-HIGGINS, AND MILLER, IN THE PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY, SERIES A. VOLUME 246 (1954) PAGES 401-449, WHICH IS A TECHNICAL ACCOUNT OF THEM, PLUS EXCELLENT DRAWINGS OF EACH OF THEM. YOU MIGHT ALSO ENJOY <u>POLYHEDRON MODELS</u> AND <u>DUAL MODELS</u> BY MAGNUS WENNINGER, WHICH HAVE PHOTOS OF MODELS FOR THE UNIFORM POLYHEDRA AND THEIR DUALS, PLUS INSTRUCTIONS ABOUT HOW TO MAKE YOUR OWN MODELS.





THE GREAT RETROSNUB ICOSIDODECAHEDRON HAS 12 PENTAGRAMS AND 80 TRIANGLES IN PLANES PARALLEL TO THE FACES OF THE SNUB DODECAHEDRON. SAY THE DISTANCE FROM THE CENTER TO A FACE IS i, THEN FOR THE SNUB DODECAHEDRON, $\frac{i_T}{i_P} = 1.04855012$

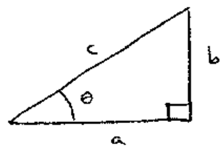
BUT FOR THIS, $\frac{i_T}{i_P} = .226125894$

ALSO SAY THIS SHAPE HAS EDGE LENGTH = 1, THEN IT FITS INTO A SPHERE OF DIAMETER

$D = 1.160003009$

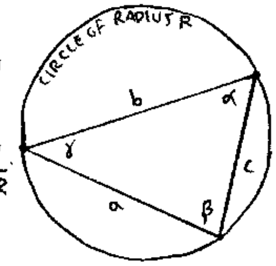
I ADVISE YOU TO EAT A HEARTY BREAKFAST BEFORE ATTEMPTING TO BUILD THIS.

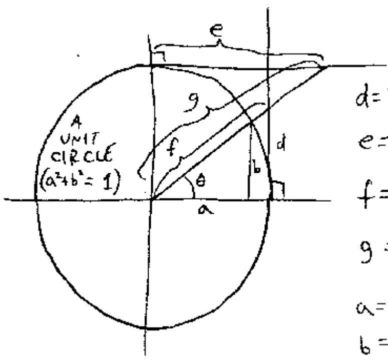
USEFUL MATH THINGS



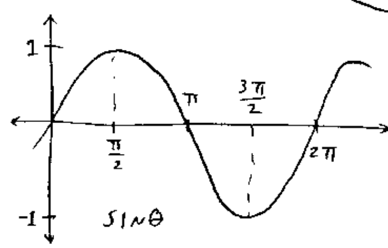
$$a^2+b^2=c^2$$
$$\cos\theta = \frac{a}{c}$$
$$\sin\theta = \frac{b}{c}$$
$$\tan\theta = \frac{b}{a}$$

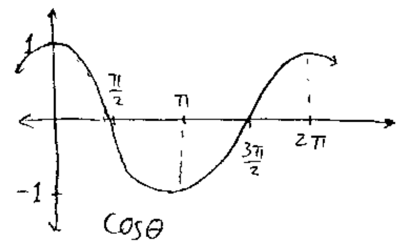
$$\alpha+\beta+\gamma = 180^\circ$$
$$180^\circ = \pi \text{ RADIANS}$$
$$c^2 = a^2+b^2-2ab\cos\gamma$$
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$$



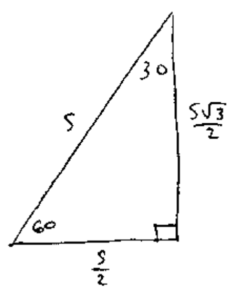


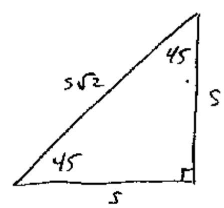
$$d = \tan\theta$$
$$e = \frac{1}{\tan\theta}$$
$$f = \frac{1}{\cos\theta}$$
$$g = \frac{1}{\sin\theta}$$
$$a = \cos\theta$$
$$b = \sin\theta$$





$$\cos^2\theta+\sin^2\theta = 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$2\sin^2\theta = 1-\cos 2\theta$$
$$2\cos^2\theta = 1+\cos 2\theta$$
$$1+\tan^2\theta = \frac{1}{\cos^2\theta}$$
$$\sin\theta = \cos(90-\theta)$$
$$\sin\theta = -\sin(-\theta)$$
$$\cos\theta = \cos(-\theta)$$





$$\cos 0 = 1 \qquad \sin 0 = 0$$
$$\cos 90 = 0 \qquad \sin 90 = 1$$
$$\cos 30 = \sin 60 = \frac{\sqrt{3}}{2}$$
$$\cos 60 = \sin 30 = \frac{1}{2}$$
$$\tan 60 = \sqrt{3}$$
$$\tan 45 = 1$$
$$\cos 45 = \sin 45 = \frac{1}{\sqrt{2}}$$
$$(\cos 72)(\cos 36) = \frac{1}{4}$$
$$(\tan 72)(\tan 36) = \sqrt{5}$$

α	ALPHA	ν	NU
β	BETA	ξ	XI
γ	GAMMA	o	OMICRON
δ	DELTA	π	PI
ϵ	EPSILON	ρ	RHO
ζ	ZETA	σ	SIGMA
η	ETA	τ	TAU
θ	THETA	υ	UPSILON
ι	IOTA	ϕ	PHI
κ	KAPPA	χ	CHI
λ	LAMDA	ψ	PSI
μ	MU	ω	OMEGA